Cooperative Sequential Compressed Spectrum Sensing over Wide Spectrum Band

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***Abstract*—Cognitive radio (CR) techniques promise to significantly increase the available spectrum thus wireless bandwidth. With the increase of spectrum allowed for CR, it is critical and challenging to perform efficient wideband sensing. We propose an integrated sequential wideband sensing framework which concurrently exploits sequential detection and compressed sensing (CS) techniques for more accurate and lower cost spectrum sensing. First, to ensure more timely spectrum detection while avoiding the high overhead involved in periodic recovery of CS signals, we design a CS-based sequential wideband detection scheme to effectively detect the PU activities in the wideband of interest. Second, to further identify the sub-channels occupied, we exploit joint sparsity of the signals among neighboring users to achieve efficient cooperative wideband sensing. Our performance evaluations demonstrate that our proposed scheme can outperform other peer schemes significantly in terms of the detection delay, detection accuracy, sensing overhead and sensing accuracy.**

***Index Terms*—cognitive radio; sequential detection; wideband sensing; compressed sensing; cooperative sensing.**

I. Introduction

Cognitive radio (CR) is attracting growing interests due to its capability of intelligently and dynamically identifying and exploiting the spectrum holes to improve the spectral usage efficiency [1], [2]. A core function of CR (or secondary user, SU) is to sense the spectrum and detect the presence or absence of the primary users (PUs). Many studies have been done to improve the effectiveness of spectrum sensing. Sequential analysis has been exploited with a slotted sensing structure [11] or with periodic spectrum sensing [14] for better sensing performances.

Spectrum sensing for wideband becomes increasingly important for CRs to obtain a "wider" view of the spectrum. It enables a CR to find spectrum resources more flexibly and quickly, and also allows a CR to transmit data at higher rate with more spectral resources available. Wideband sensing, however, is challenging. A wideband can be generally divided into sub-bands or sub-channels, whose occupancy status (i.e. occupied by PUs or not) can be determined through sensing. One possible way is to sense all the narrow sub-channels one by one, and there are many existing studies on the scheduling of channel sensing order [10]. Although applicable for a band with a limited number of sub-channels, for a wideband with extremely large number of sub-channels, sensing each channel one by one will bring large overhead and delay. Alternatively, CRs can sense the wideband directly with some high-end wideband components equipped at higher cost, e.g., wideband attenna and radio frequency (RF) front-end, high-speed analog-to-digital converter (ADC) .

In order to avoid the use of costly high-speed ADC in wideband sensing, compressed sensing (CS) theory [3], [4] has been exploited to reduce the number of samples required [15], [16]. However, due to the higher computational complexity for CS recovery in wideband sensing, it would be very expensive to directly apply these CS methods to perform periodic sensing. On the other hand, simply making sensing decision based on data collected within one time period is also prone to failure caused by low SNR and in the presence of noise. The spectrum sensing becomes even harder when a user receives weak signals as a result of channel fading. Although cooperative sensing may help overcome this problem, simply applying cooperative wideband sensing using samples from a large number of users would involve high computational complexity.

In this work, we consider a CR network with multiple users. To address the issues above, we propose a *cooperative sequential compressive sensing* framework which incorporates two major steps, *wideband signal occupancy detection* to detect the presence of PU in the wideband of interest, and *cooperative wideband compressive sensing* to determine which sub-bands are actually occupied in the wideband by obtaining the wideband power spectrum. Different from conventional CS-based wideband sensing, the first step takes advantage of both sequential analysis and compressive sampling to first detect if there exist PU signals in a wide band without need of complex signal reconstruction. In contrast to conventional cooperative spectrum sensing which simply exploits the diversity of user sensing data to improve the sensing performances, the second step takes advantage of *joint sparsity* of signals from multiple SUs to further reduce the sampling rate while improving the sensing accuracy. The major contributions of our work are as follows:

- We incorporate the compressed sensing technique into

the sequential periodic wideband detection, taking advantage of both techniques for accurate and low-overhead PU detection. Specifically, we perform sequential analysis [17] based on sub-Nyquist samples directly without incurring high CS recovery overhead, and exploit sequential detection to improve the detection performance.
- We exploit the intra-signal temporal correlation and the inter-signal spatial correlation to perform more efficient cooperative compressed spectrum sensing. Instead of simply performing wideband sensing for each user, we employ *joint sparsity model* to jointly recover the wideband power spectrum sensed by cooperative users which further reduces the sampling rate and CS reconstruction overhead.
- We perform extensive simulations to validate and demonstrate the major advantages of our design.

The rest of this paper is organized as follows. The next section gives an overview of related work. We then present some backgrounds in Section III and the system model in Section IV. We describe our scheme on sequential wideband detection with compressive sampling in Section V, and cooperative wideband compressive sensing in Section VI. In Section VII, we provide the simulation results with various discussions. The paper is concluded in Section VIII.

II. Related Work

Sequential analysis [17] has been applied in spectrum sensing to attain better performances such as shorter latency and more precise decision. Kim *et al.* in [21] and Min *et al.* in [22] proposed to apply sequential spectrum sensing in CRNs. In [14], [20], the authors show that scheduling periodic sequential sensing helps to improve the spectrum sensing performances. Some studies, such as [23] and [24], have taken into account change detection for cognitive radios, while we also propose a change detection scheme to improve the sequential detection performance for wideband.

Different from existing efforts, the focus of this paper is on effective detection of the activities of legacy wireless systems over a wide spectrum. The sequential detection is only applied over sparse samples of signals (rather than Nyquist samples) to facilitate low cost coarse signal monitoring, before we determine the actual sub-channels occupied by the primary signals.

Compressed sensing (CS) is a useful tool for wideband spectrum sensing and analysis. Tian *et al.* [16] developed CS techniques tailored for wideband sensing to identify spectrum holes, where sub-Nyquist samples are used along with a wavelet-based edge detector. Similarly, in [15], [25]–[27], various wideband spectrum sensing schemes based on CS are proposed. There are also some references discussing cooperative wideband sensing based on CS. For example, in [28] [29] distributed compressed wideband sensing schemes are proposed.

The goal of this work is to enable efficient periodic cooperative wideband sensing. Different from the literature work, to reduce the sensing overhead and improve the sensing performance, our scheme is divided into two steps: (a) We exploit CS-based sequential detection (without compressed sensing reconstruction) to first detect PU's presence in the wideband; (b) We employ joint sparsity models to recover the wideband status from the signals perceived in the sequential analysis, which takes advantage of the correlation of user samples to further reduce the samples needed and speed up the detection process.

III. Preliminaries

Before we present our detailed sensing algorithms, we first provide some background knowledge on wideband sensing and compressed sensing.

A. Wideband Sensing

For traditional channel detection, the two hypotheses regarding the state of the channel are:
- H_0: The channel is available (the PU is absent);
- H_1: The channel is occupied by the PU,

A wideband can usually be divided into sub-bands/sub-channels. In the example of Figure 1, a wide spectrum band is shown to range from 0 to W(Hz) and equally divided into J sub-bands with the bandwidth of each being W/J (Hz). Depending on the spectrum band of interest, the band may not start from 0 Hz. In addition, sub-bands are not necessarily of the same bandwidth. The power spectrum characteristics across the wideband can be used to indicate the status of each sub-band, e.g. by thresholding the power amplitude in each sub-bands. In this framework, all the PUs within the wideband can be regarded as a PU group, which occupies part of the sub-channels in the wideband. Throughout this paper, we will interchangeably use sub-band/sub-channel.

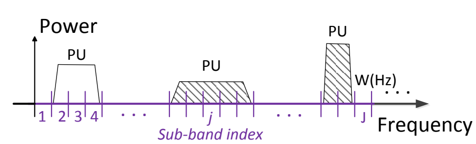

Fig. 1. Frequency division for wideband CRs. CR can employ OFDM-like techniques that divide the wideband spectrum into J sub-bands.

For wideband sensing, a big challenge is that the required Nyqusit sampling rate can be fairly large, which would lead to a high cost to ADC elements and also brings more processing overhead. For example, considering a 0∼500 MHz wideband, the Nyquist sampling rate will be 1 GHz. This motivates us to exploit compressed sensing to significantly reduce the required sampling rate for wideband sensing.

B. Compressed Sensing

The main idea of compressed sensing is to take advantage of the sparsity within the signal to significantly reduce the sampling rate. An N-dimensional signal $\mathbf{d}$ is considered to be K-sparse in a domain (also called a dictionary matrix) $\mathbf{\Psi} \in \mathbb{C}^{N\times N}$ if there exists an N-dimensional vector $\mathbf{x} \in \mathbb{R}^{N\times 1}$ so that $\mathbf{d} = \mathbf{\Psi}\mathbf{x}$ and $\mathbf{x}$ has at most K non-zero entries ($K \ll N$).

If a signal $\mathbf{d}$ is measured with a matrix $\mathbf{\Phi} \in \mathbb{C}^{M\times N}$ ($M < N$), the obtained linear measurement $\mathbf{y} \in \mathbb{R}^{M\times 1}$, possibly affected by noise $\mathbf{n} \in \mathbb{R}^{M\times 1}$, is:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{x} + \mathbf{n} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{1}$$

where the *sensing matrix* $\mathbf{A} \in \mathbb{R}^{M\times N}$ is essentially the product of the *measurement matrix* and the *dictionary/basis matrix*: $\mathbf{A} = \mathbf{\Phi}\mathbf{\Psi}$.

Obviously, with $M < N$, Equation (1) is under-determined. E. J. Candès *et al.* show in [5] that the under-determined equation system can be solved if some conditions are met. One of the conditions is the number of measurements M is enough:

$$M \geq cK \log\left(\frac{N}{K}\right), \tag{2}$$

where c is a fairly small constant. Further details can be found in [5]. Given the measurements $\mathbf{y}$, the unknown sparse vector $\mathbf{x}$ can be reconstructed by solving the following convex optimization problem:

$$\min \quad \|\mathbf{x}\|_{\ell_1}, \quad \text{s.t.} \quad \|\mathbf{\Phi}\mathbf{d} - \mathbf{y}\|_{\ell_2} \leq \epsilon, \mathbf{d} = \mathbf{\Psi}\mathbf{x} \tag{3}$$

where the parameter ϵ is the bound of the error caused by noise $\mathbf{n}$, ℓ_p means the ℓ_p-norm ($p = 1, 2, ...$). The solution can also be expressed as:

$$\hat{\mathbf{x}} = \underset{\mathbf{u}:\ \|\mathbf{y}-\mathbf{A}\mathbf{u}\|_{\ell_2}\leq\epsilon}{\operatorname{argmin}} \|\mathbf{u}\|_{\ell_1}. \tag{4}$$

The signal $\mathbf{d} = \mathbf{\Psi}\mathbf{x}$ can then be recovered as $\hat{\mathbf{d}} = \mathbf{\Psi}\hat{\mathbf{x}}$.

In addition to the convex optimization approach to solve the problem above, such as ℓ_1 minimization [6], there exist several iterative/greedy algorithms such as Cosamp [13]. Such convex or greedy approaches are generally called reconstruction algorithms.

IV. System Model

We consider a general CR network with a set of CR nodes, each can sense the wide spectrum band to find some unoccupied spectrum channels to transmit data. PUs generally alternate between a period of activity and a period of idle time, and it is most critical to detect the change of the channel occupancy state.

In this section, we first introduce the basic operational model, we then introduce the infrastructure for the periodic sensing and the framework for compressed detection and sensing over wideband.

A. Basic Operational Model

In order to timely detect the channel condition change, each user will periodically sense the wide spectrum band. As CS recovery would involve a high computational complexity, reconstructing the spectrum signals in each sensing period is costly. Instead, each SU first sub-samples the wideband spectrum it monitors, and then performs the sequential detection based on the sub-sampled data. If the spectrum is detected to have PU activities, CS recovery is further called for to detect the occupancy conditions of sub-channels in a wide band.

Due to fading, different users may receive signals in different conditions. If a user receives very weak signals as a result of severe channel condition, it would be difficult for the user to make detection decision. It would also be hard for the user to correctly identify the sub-channels occupied through CS reconstruction. In this case, a user with weak channel condition can request a collaboration from its neighbors to make joint detection decision, and neighboring CS nodes can collaborate in performing the joint CS reconstruction to identify the sub-channels occupied at lower cost.

B. Periodic Sensing Structure

Fig. 2 illustrates the periodic channel sensing structure, where the channel detection time (CDT) is the maximum allowed time for a sensing decision to be made. A CDT usually consists of multiple sensing-transmission periods, each being called a *sensing period* T_p. In this work, as in 802.22 WRAN standard, the *sensing time* T_s is fixed, e.g., 1 ms, and T_p may only take values that are multiples of a MAC frame size 10 ms due to many higher-layer concerns such as synchronization.

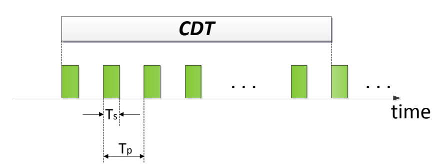

Fig. 2. Channel detection time CDT, sensing period T_p, and sensing time T_s

The *detection overhead* (R_{do}) describes the proportion of time dedicated to the PU detection task and is defined as the ratio between T_s and T_p, i.e., $R_{do} = T_s/T_p$. In this work, T_s (1 ms) is rather short compared to T_p ($k \cdot 10$ ms), so the detection overhead is at most 10% for satisfactory SU communication performance. Scheduling of sequential detection will have a significant influence on the detection overhead.

C. Compressed Sensing for Wideband Detection and Wideband Sensing

If a CR samples the signal of interest for a duration of T_s, the continuous signal received at the RF front-end of CR, i.e., $d_c(t)$, is composed of PUs' signals and background noise. By using a certain sampling rate f_N

over the sensing time T_s, we could obtain a discrete time sequence $d[n] = d_c(\frac{n}{f_N}),\ n = 0, 1, \cdots, N-1$, in a vector form $\mathbf{d} \in \mathbb{C}^{N\times 1}$. Here, $N = T_s f_N$ is usually chosen to be a positive integer.

To reduce the cost of wideband sensing, in our framework, SU's detector collects the signal at a certain sub-Nyquist sampling rate, $f_{sub} < f_{nyq}$, with f_{nyq} being the Nyquist sampling rate. Different from periodic Nyquist sampling, the samples are taken randomly based on a measurement matrix $\mathbf{\Phi}$ whose dimension is $M \times N$ ($M < N$), where $M = f_{sub}T_s$ and $N = f_{nyq}T_s$ denote the number of sub-Nyquist samples and Nyquist samples, respectively.

Mathematiclaly, if there is any PU signal within the wideband of our interest, the sub-sample vector in T_s for a certain SU will be expressed as

$$\begin{aligned} \mathbf{y} &= \mathbf{\Phi}(\mathbf{d} + \mathbf{n}') = \mathbf{\Phi}\mathbf{d} + \mathbf{n} \\ &= \mathbf{\Phi}\mathbf{\Psi}\mathbf{x} + \mathbf{n} = \mathbf{A}\mathbf{x} + \mathbf{n}, \end{aligned} \tag{5}$$

where the sub-Nyquist measurements are $\mathbf{y} \in \mathbb{R}^{M\times 1}$, the sparse vector in Fourier spectrum domain $\mathbf{x} \in \mathbb{R}^{N\times 1}$, the additive noise in the wideband $\mathbf{n}'$, the sampled noise $\mathbf{n} \in \mathbb{R}^{M\times 1}$, and the sensing matrix $\mathbf{A} \in \mathbb{R}^{M\times N}$.

It is mentioned in [9], [19] that if the signal's spectrum vector $\mathbf{x} = \mathbf{\Psi}^{-1}\mathbf{d}$ is sparse ($\mathbf{\Psi}^{-1}$ is the Discrete Fourier Transform), then $\mathbf{\Phi} = \mathbf{A}\mathbf{\Psi}^{-1}$ is essentially an $M \times N$ random sampling matrix constructed by selecting M rows independently and uniformly from an $N \times N$ identity matrix $\mathbf{I}$. This measurement matrix $\mathbf{\Phi}$ can be trivially implemented by pseudo-randomly sub-sampling the original signal $\mathbf{d}$. As we can adopt inverse DFT matrix as the sparse dictionary $\mathbf{\Psi}$, in our framework, the measurement matrix will be reflected by sub-Nyquist sampling. For time domain signals with length N, this measurement process corresponds to smaller sampling numbers $M < N$. If the spectral sparsity level K of $\mathbf{x}$ is known, we can choose the number of measurements M to secure the quality of spectral recovery, as expressed in Equation (2).

V. Spectrum Detection based on Sequential Compressed-Sensing

In the cooperative detection network we consider, each SU periodically sub-samples the wideband signal based on the theory of compressed sensing, and detects the spectrum activity through group-based compressive sequential detection. If a user receives very weak signals as a result of severe channel conditions such as fading, it can send a request to its neighbors to seek collaborations, and perform collaborative sequential detection based on responses from neighboring users.

A. Cooperative Grouped-Compressed-Data SPRT

Wald's Sequential Probability Ratio Test (SPRT) [17] is a classic sequential detection methodology which puts each sample into the sequential test. We consider a cooperative *grouped-compressed-data SPRT* (GCD-SPRT), which departures from the conventional SPRT in four perspectives: 1) Instead of taking samples at Nyquist rate, in each T_s, an SU randomly takes much smaller number of samples based on the CS theory; 2) Data samples within each T_s are grouped into a "super-sample" to avoid the complexity of processing each sample and more importantly to reduce the effect of short-term channel randomness; 3) The sub-sampling within T_s and the use of super samples in different time periods can exploit the temporal data redundancy and time diversity to reduce the total number of samples; 4) Besides making the sequential detection at each SU, a SU receiving weak signals could request a cooperation from its neighbors to fuse their data along with its own, taking advantage of spatial diversity to make faster detection decision. Cooperative GCD-SPRT can be performed at an SU as follows:

Step 1: Calculate the power $z(\mathbf{y})$ from M compressed samples.

If there is any PU signal within the wideband of our interest, the sub-sample vector will be expressed as in Equation (14). After a sensing block T_s, the normalized power of M sub-Nyquist samples contained within is

$$z(\mathbf{y}) = \frac{\sum_{i=1}^{M} y_i^2}{T_s} = \frac{f_{sub}}{M}\sum_{i=1}^{M} y_i^2, \tag{6}$$

where y_i denotes the individual samples within T_s, and $M = f_{sub}T_s$.

In practice, the number of samples taken within a single T_s is fairly large. For example, for $T_s = 1$ ms and a $0 \sim$ 500MHz wideband, the Nyquist sample number will be $N = 10^6$; and even if we perform sub-sampling with 1/10 of the Nyquist sampling rate, we will have 10^5 sub-Nyquist samples. With the Law of Large Numbers ($M \gg 10$) and Central Limit Theorem, we have the average signal within a T_s approximating Gaussian regardless of the original distribution of the PU signal, that is, $z(\mathbf{y}) \overset{i.i.d.}{\sim}$

$$\begin{cases} H_0 : \mathcal{N}(f_{sub}P_n, \frac{(f_{sub}P_n)^2}{M}), \\ H_1 : \mathcal{N}(f_{sub}P_n(1 + SNR), \frac{(f_{sub}P_n)^2(1+SNR)^2)}{M}), \end{cases} \tag{7}$$

which can be similarly derived from the results in [14]. Here, SNR is defined as the ratio between the nominal signal power P and local noise floor $\sigma^2 = P_n W$, where P_n is the noise power spectral density (PSD) and W is the wideband's bandwidth.

Step 2: Derive the test statistic $T(z(\mathbf{y}))$ for each group.

The log-likelihood ratio (LLR) of the power sample is calculated as

$$T(z(\mathbf{y})) = \ln \frac{f_1(z(\mathbf{y}))}{f_0(z(\mathbf{y}))}, \tag{8}$$

where $f_0(\cdot)$ and $f_1(\cdot)$ are the pdfs under H_0 and H_1, respectively, as indicated in Eq. (7). For ease of presentation, we will simply refer to $z(\mathbf{y})$ as z.

Step 3: Accumulate the test statistics $T(z)$ across groups to obtain the aggregate test statistic $\mathcal{T}$

As we accumulate $T(z_k)$ $(k = 1, 2, ...)$ sequentially, the aggregate test statistic up to the s-th group is

$$\mathcal{T}_s = \sum_{k=1}^{s} T(z_k) = \sum_{k=1}^{s} \ln \frac{f_1(z_k)}{f_0(z_k)}. \tag{9}$$

Step 4: On demand cooperation with other users

If a user cannot make timely detection decision due to its weak signal conditions, it can request its neighbors to collaborate. In response, an SU q can share its own aggregated test statistic $\mathcal{T}_{s_q}^{q}$. Assume the user receives responses from $Q-1$ cooperating users, its can form the accumulated cooperative test statistic as:

$$\mathcal{T}_s' = \mathcal{T}_s + \sum_{q=1}^{Q-1} \mathcal{T}_{s_q}^{q} \tag{10}$$

Step 5: Make detection decision

Compare the accumulated cooperative test statistic in Equation (10) against two constant thresholds A and B.

The two decision thresholds are chosen in Wald's SPRT as:

$$A = \ln \frac{p_{MD}}{1 - p_{FA}}, \text{ and } B = \ln \frac{1 - p_{MD}}{p_{FA}}. \tag{11}$$

The decision rule for the SU is designed as:

- if $\mathcal{T}_s' > B$, it decides that the PU has reclaimed the channel.
- if $\mathcal{T}_s' < A$, it decides that the channel is still available;
- otherwise, it goes to **Step 1** to continue sampling another group of power data, cooperating with other users, and updating $\mathcal{T}'_{s+1}$ using Eq. (10).

The *stopping time* S for an SU is defined as the minimum number of groups of LLR statistic (of the user itself) needed until one of the two decision thresholds is first crossed; that is,

$$S = \min\{s : \text{ either } \mathcal{T}_s' < A \text{ or } \mathcal{T}_s' > B\}. \tag{12}$$

If S is small, it means that the SU can make the detection decision (PU is present or not) faster. This helps an SU to evacuate the channels timely upon the return of PU, or spend more time for data transmission upon the detection of channel idle.

B. Quick Detection of Channel State Changes

It is important to quickly detect the "change point" where the wideband state shifts from H_0 to H_1 due to PU reappearance or vice versa. Straight-forwardly, sensing decision can be made once in each CDT window. After a channel is detected to be idle, an SU can dedicate to transmission as shown in Fig. 3(a), similar structure without compressed sensing is given in [22]. However, if the PU reappears, the channel will not be sensed until the next CDT window, which may make the evacuation delay of the SU exceed the CDT time, the maximum delay allowed for an SU to evacuate the channel.

Instead, we consider using backward GCD-SPRT along with moving CDT window (Fig. 3 (b)), where GCD-SPRT can run *backward*, starting from the latest group of data. This helps reduce the impact of the older sensing data to more quickly detect the possible status change. In order to further speed up the change point detection, we propose an in-depth sensing method in which a CR adjusts its sensing frequency to ensure more rapid and precise detection after suspecting the possible H_0-to-H_1 transition, as in Fig. 3 (c).

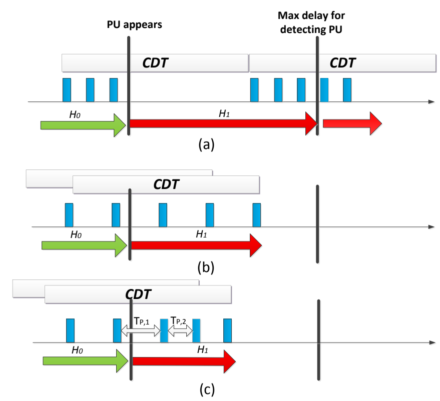

Fig. 3. Detection delay (indicated by the red arrows) with (a) forward, non-overlapping GCD-SPRT; (b) backward, overlapping GCD-SPRT; and (c) backward, overlapping GCD-SPRT with short-term T_p adjustment

It is critical to determine when an *in-depth sensing* should be triggered. We set the following criteria:

$$\begin{aligned} \mathcal{T}_c = \max\{\bar{\mathcal{T}}_{new} - \bar{\mathcal{T}}_{old}\} &\geq \delta(n_{new}\frac{B}{n} - n_{old}\frac{A}{n}) \\ &= \delta(n_{new}B - n_{old}A)/n, \end{aligned} \tag{13}$$

in which $\mathcal{T}_c$ is the test statistic that will trigger in-depth sensing; $\bar{\mathcal{T}}_{new}$ and $\bar{\mathcal{T}}_{old}$ are the summation of the newer and older test statistics in the CDT-window, n_{new} and n_{old} are the numbers of test statistics classified as newer and older, and $n = n_{new} + n_{old}$; B and A are the thresholds in Equation (11), and δ is the parameter that controls the sensitivity of SU to the shift. With a smaller δ value, SU is more sensitive to the changes in the observed data.

For the given set of test statistics in the CDT-window, the SU starts with the most recent test statistic (with the remaining sensing blocks in the window as "old") and obtains the difference; then the new test statistic set includes one more recent test statistic, with the old test statistic set losing this test statistic, so the numbers of newer test statistics and older ones are increased and decreased by one respectively. This test continues until the data from a CDT-window have all been checked. If Equation (13) is still not met, then GCD-SPRT-based sequential detection is pursued with T_p unchanged; otherwise, T_p will be changed and an in-depth sensing is performed to speed up the decision process. Choices of T_p will be discussed in the simulations.

VI. Cooperative Wideband Compressed Sensing

After detecting the existence of PU activities in a wideband, it is necessary to determine the sub-channels actually occupied so the remaining spectrum can be used by SUs for transmissions. In this work, we exploit compressive sensing to reconstruct the spectrum usage map from the sub-sampled data. This recovery can be done by individual users independently. However, if a user receives weak signals, the accuracy of spectrum reconstruction is low. Instead, we take advantage of the joint sparsity of samples from neighboring users to cooperatively recover the spectrum usage maps. This not only helps to more accurately recover the spectrum occupancy maps, but also helps to significantly reduce the total number of samples thus the sampling cost.

A. Wideband Spectrum Usage Detection with Intra-User Compressive Sensing

With samples collected from multiple time periods in sequential detection, a user can first fuse its temporal samples taking advantage of the time diversity for better recovery of the spectrum signals.

As the user senses the same spectrum over time, the basis $\mathbf{\Psi}$ for a signal to project to remains the same. If an SU adopts the same measurement matrix $\mathbf{\Phi}$ before a local sequential detection decision is made, we can average the compressed readings from all time periods:

$$\begin{aligned} \bar{\mathbf{y}} &= \mathbf{\Phi}(\bar{\mathbf{d}} + \bar{\mathbf{n}'}) = \mathbf{\Phi}\bar{\mathbf{d}} + \mathbf{n} \\ &= \mathbf{\Phi}\mathbf{\Psi}\bar{\mathbf{x}} + \bar{\mathbf{n}} = \mathbf{A}\bar{\mathbf{x}} + \bar{\mathbf{n}}, \end{aligned} \tag{14}$$

where we have the average values for the sub-Nyquist measurements $\bar{\mathbf{y}} \in \mathbb{R}^{M\times 1}$, sparse vector in Fourier spectrum domain $\bar{\mathbf{x}} \in \mathbb{R}^{N\times 1}$, additive noise in the wideband $\bar{\mathbf{n}'}$, the sampled noise $\bar{\mathbf{n}} \in \mathbb{R}^{M\times 1}$, and the sensing matrix $\mathbf{A} \in \mathbb{R}^{M\times N}$. Each user can individually recover its $\bar{\mathbf{x}}$ by solving the optimization problem in (3).

Next, we will investigate the benefit of exploiting cooperative compressive sensing. For simplicity, we will now disregard the averaging mark and denote this averaged result for the q-th secondary user as i.e., $\mathbf{y}_q, \mathbf{x}_q$ etc. $q \in \{1, 2, ..., Q\}$. Q is the number of SUs.

B. Detection of Wideband Spectrum Occupancy with Inter-User Cooperative Compressive Sensing

Each user can independently perform wideband spectrum sensing with the number of CS samples sufficient to reconstruct the spectrum map. When the channel condition is not good, the number of samples required and the duration of sensing would be high. When the received signals are very weak, there is also a possibility not being able to recover the spectrum signals. In the case of the existence of multiple users in a neighborhood, the users can collaborate in spectrum sensing to improve the sensing quality as well as reduce the sensing time and samples. In a dedicated control channel, every SU can share its average signal samples with nearby users that are within its transmission range.

With data from Q users, a straight-forward way of cooperation is to concatenate samples from all users and process together using a super CS matrix. However, this may introduce a high computational overhead. As samples from neighboring SUs may have strong spatial correlation, the redundant samples will not efficiently contribute to the CS recovery process. To avoid this issue, we will exploit the Joint Sparsity Model 1 (JSM-1) [7] to significantly reduce the measurement requirement and reconstruction overhead.

Due to the spatial correlation, the readings at nearby users may have common factors, generally introduced by global conditions such as the PU group activities. Besides, the readings at each user also have localized factors, introduced by its individual condition, such as spatial location and local noise. The actual PU signal (before sub-Nyquist sampling) received at a secondary user q, $q \in \{1, 2, ..., Q\}$, can be expressed as:

$$\mathbf{d}_q = \mathbf{s}_c + \mathbf{s}_q, q \in \{1, 2, ..., Q\}, \tag{15}$$

where

$$\mathbf{s}_c = \mathbf{\Psi}\mathbf{x}_c, \|\mathbf{x}_c\|_0 = K_c, \mathbf{s}_q = \mathbf{\Psi}\mathbf{x}_q, \|\mathbf{x}_q\|_0 = K_q, \tag{16}$$

where $\mathbf{s}_c$ is the *sparse-common* component that is common to all of the $\mathbf{d}_q$ and has the sparsity K_c in the basis $\mathbf{\Psi}$. $\mathbf{s}_q$ is the *sparse-innovations* (unique portions) of the $\mathbf{d}_q$ and each has the sparsity K_q in the same basis.

We can see the benefit of exploiting the joint sparsity in a simple case of $Q = 2$ users, when a node collaborates with its most adjacent node [12]. If CS theory is directly employed, we may need the number of measurements in the order of $c(K_c+K_1)$ to reconstruct $\mathbf{d}_1$ and $c(K_c+K_2)$ to reconstruct $\mathbf{d}_2$, respectively. To recover the two signals together, we only need $c(K_c + K_1 + K_2)$ measurements.

1) Joint Recovery of Spectrum among SUs: For cooperative recovery of signals from Q users, let $\Lambda := \{1, 2, \dots, Q\}$ denote the set of indices for the Q signals in the ensemble. Denote the *signal* in the ensemble by $\mathbf{d}_q \in \mathbb{R}^N$, which is sparse in basis $\mathbf{\Psi}$, with $q \in \Lambda$.

To compactly represent the signal and measurement ensembles, we denote $\widetilde{M} = \sum_{q\in\Lambda} M_q$ and define $\mathbf{X}$, $\mathbf{D}$, $\mathbf{Y}$, and $\widetilde{\mathbf{\Phi}}$ as

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_c \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_Q \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_Q \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_Q \end{bmatrix}, \tag{17}$$

and

$$\widetilde{\mathbf{\Phi}} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{\Phi}_Q \end{bmatrix}, \tag{18}$$

$$\widetilde{\mathbf{\Psi}} = \begin{bmatrix} \mathbf{\Psi} & \mathbf{\Psi} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{\Psi} & \mathbf{0} & \mathbf{\Psi} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Psi} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{\Psi} \end{bmatrix}, \tag{19}$$

Using the structured $\widetilde{\mathbf{\Psi}}$, we can represent $\mathbf{D}$ sparsely using vector $\mathbf{X}$, which contains $K_c + \sum_{q=1}^{Q} K_q$ non-zero elements, to obtain $\mathbf{D} = \widetilde{\mathbf{\Psi}}\mathbf{X}$. We then have $\mathbf{Y} = \widetilde{\mathbf{\Phi}}\widetilde{\mathbf{\Psi}}\mathbf{X} = \widetilde{\mathbf{A}}\mathbf{X}$. With sufficient measurements, we can recover the vector $\mathbf{X}$, and thus $\mathbf{D}$ (all $\mathbf{d}_q$) and wideband status ($\mathbf{x}_c + \mathbf{x}_q$), by solving the following problem:

$$\min \quad \|\mathbf{X}\|_{\ell_1} \tag{20a}$$

$$\text{s.t.}\mathbf{Y} = \widetilde{\mathbf{\Psi}}\widetilde{\mathbf{\Phi}}\mathbf{X}, \tag{20b}$$

which can be solved with a single linear program.

The fusion can be done each user or a cluster head. After getting the compressed readings $\mathbf{y}_q$ from all users, a fusion node can form new matrices as in (17) (18) (19) and solve the problem as in (20). After getting the estimated wideband status ($\mathbf{x}_c + \mathbf{x}_q$) for each user, it needs to fuse the results in order to get a more accurate cooperative wideband sensing result.

2) Fusion of Wideband Spectrum Maps: With the spectrum usage condition observed from each user, we can fuse the results with two candidate schemes:

(a) **Soft Fusion.** Power spectrum usage maps recovered by all users are averaged to get a new map, and each sub-band value is compared with an energy threshold to determine which sub-bands are occupied. We will adopt soft fusion in the simulations.

(b) **Hard Fusion.** The spectrum energy map from each user is applied to determine which sub-bands are occupied individually, and the resulted binary spectrum maps (where the occupied sub-bands marked as "1" and the idle ones marked as "0") are merged by OR rule (other options are AND rule, majority rule, weighted combing rule and so on).

VII. Simulations and Results

In this section, we conduct MATLAB simulation studies to demonstrate the performance of our design. We also compare our scheme with other peer schemes to show the advantages of our work.

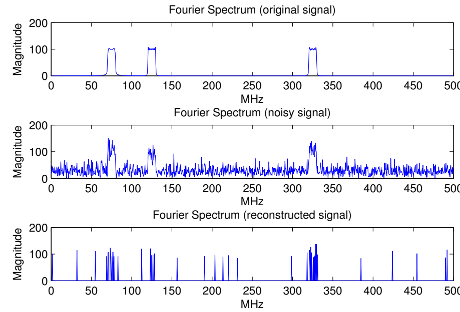

Fig. 4. Compressed sensing recovery under $SNR = -5dB$

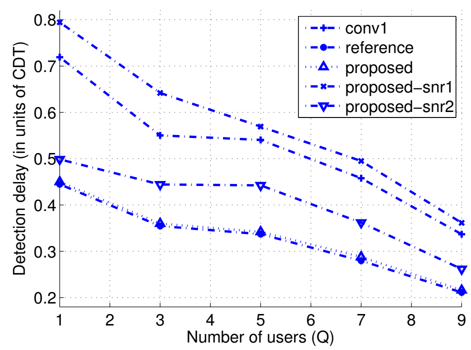

Fig. 5. Detection delay

A. Simulation Settings

1) System Setup: We consider a wideband of 500 MHz, which can be virtually divided into 50 sub-bands, each occupying 10 MHz. The Nyquist sampling rate is 1 GHz. A PU group signal is a wideband signal that spreads over the wideband, but may only occupy a small portion of the wideband, i.e., the number of occupied sub-bands is much smaller than the total number of sub-bands monitored. The noise is assumed to be circular complex AWGN, i.e., $\mathbf{n} \sim \mathcal{N}(0, \eta^2)$. SNR values will be given in specific tests.

For the periodic sensing model, we set the sensing block duration $T_s = 20\mu$s, with channel detection time $CDT = 40$ms and the required $P_{FA} = P_{MD} = 0.01$. Rather than using the Nyquist sampling rate $f_{nyq} = 1$ GHz, we adopt the sub-Nyquist sampling rate $f_{sub} = 0.25$ GHz. The number of compressed samples in a sensing block T_s is $M = f_{sub}T_s = 5000$, whereas the Nyquist number $N = f_{nyq}T_s = 20,000$. An example of the wideband signal spectrum, noisy spectrum and recovered spectrum using CS is presented in Fig. 4, where the SNR is -5 dB. The wideband signal has three occupied sub-bands that have center frequencies of 75, 125 and 225 MHz respectively and each sub-band has a bandwidth of 10 MHz.

We also set the MAC frame size $FS = 200\mu$s. For the sensing period T_p, we adopt the settings similar to those in [14].

Throughout simulations, we use ℓ_1-magic as the basic reconstruction algorithm [6].

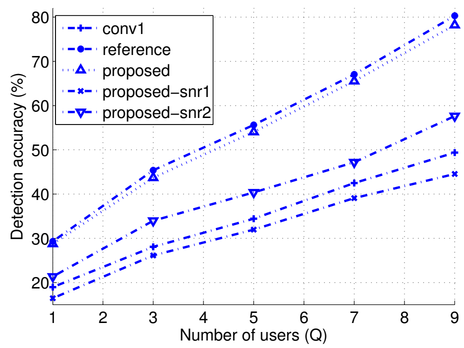

Fig. 6. Detection accuracy

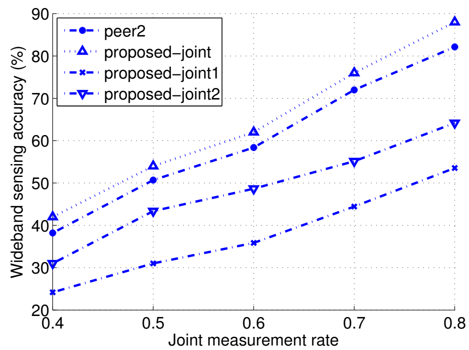

Fig. 7. Joint measurement rate reduction

B. Performance and Analysis

The peer schemes we will compare in our simulations are summarized in Table I. "reference" is the scheme with proposed sequential detection but with Nyquist sampling (without using CS). We use "reference" as a benchmark to evaluate the performance of using sub-sampling with our proposed scheme. "conv1" uses conventional non-overlapping forward SPRT without CS. "conv2" reconstructs signals every T_p in order to use recovered signals to perform sequential detection and identify the actual spectrum channel occupancy. In collaborative sensing case, "conv2" concatenates the signals from multiple users to form a super matrix for further reconstruction. Table I also gives the default SNR for each scheme (if not otherwise stated), for group 1 schemes, the default *SNR*$= -18.8$dB; for group 2, the default *SNR*$= -5$dB.

TABLE I
PEER SCHEMES COMPARISON

Group 1: default *SNR*$= -18.8$dB
"proposed": $\delta = 2$, $T_p^{new} \leftarrow 2FS$
"proposed-snr1" : proposed with *SNR*$= -20.8$dB
"proposed-snr2" : proposed with *SNR*$= -22.8$dB
"conv1" : non-overlapping forward SPRT w/o CS, see Fig. 3 (a) and [22]
"reference" : proposed sequential w/o CS, i.e., with Nyquist sampling

Group 2: default *SNR*$= -5$dB
"proposed-joint": proposed joint recovery among SUs
"proposed-joint1" : proposed with *SNR*$= -10$dB
"proposed-joint2" : proposed with *SNR*$= -15$dB
"conv2" : with CS, reconstruct signals each T_p for sequential detection
"peer2" : with CS, concatenated signal reconstruction, see [28]

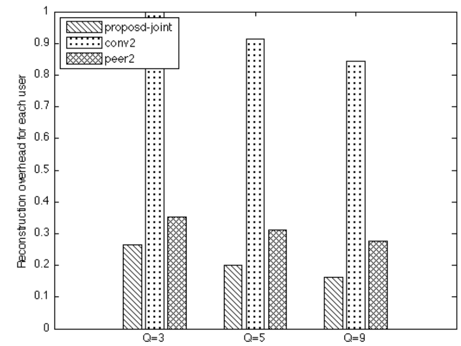

Fig. 8. Reconstruction overhead comparison.

The performance we compare include detection delay, detection accuracy, sub-band sensing accuracy and reconstruction overhead.

1) Detection Delay: The detection delay is defined as the time used by the user to make a decision. In Fig. 5, as expected, detection delay for all schemes reduce as the number of users increases. This clearly indicates the benefit of cooperative sensing. The detection delay is larger when the SNR is smaller. Compared to "conv1", our proposed scheduling of GCD-SPRT scheme can achieve much shorter delay to make a decision, thus accelerating the detection process, the delay reduction is up to 1/3.

As expected, we observe that the performances of "proposed" and "reference" are very similar, which indicates CS-based sequential detection can maintain comparable performance while reducing the number of samples required by Nyquist sampling. Performances of the proposed scheme under two other SNRs are also given.

2) Detection Accuracy: The detection accuracy is determined based on the probability of successfully detecting present PU activities in the wideband. As shown in Fig. 6, although only 1/4 of the samples are used, our proposed scheduled GCD-SPRT scheme can achieve similar performance as that not using the sub-sampling ("reference"). Compared to "conv1", GCD-SPRT achieves up to 60% higher accuracy.

We also see that when the number of users (Q) increases, the detection accuracy improves more rapidly, which implies the advantages of cooperation among SUs.

3) Wideband Sensing Accuracy: The measurement rate indicates how many measurements are used in sub-Nyquist sampling, which is calculated as the ratio of actual number of sub-Nyquist measurements over the original length (Nyquist) of the signal. Wideband sensing accuracy implies the accuracy of sub-band sensing in the wideband of interest, which is defined as the number of successfully identified (PU occupied or not) sub-channel over the total number of channels in the wideband.

From Fig. 7 it is obvious that as the measurement rate becomes larger, i.e., more measurements in the sub-sampling process, the wideband sensing (each sub-band is occupied or not) accuracy increases. This agrees with CS theory.

We see that at the same measurement rate, our scheme outperforms "peer2" because it can achieve better sensing accuracy, which shows the advantages of joint CS reconstruction. The performances of the proposed joint scheme under two other SNRs are also given in Fig. 7.

If we draw a horizontal line (measurement rate) at a certain vertical value (sensing accuracy), we can see how many measurements are needed by each scheme to achieve that sensing accuracy. We further observe that compared to "peer2" that uses long concatenated signal to perform reconstruction, the joint recovery scheme can actually reduce the requirement for measurement rate under the same requirement of wideband sensing accuracy. For example, at sensing accuracy 50%, the proposed joint recovery scheme can reduce the measurement rate by 20% compared to"peer2". The reason is that we exploit the temporal and spatial correlations in multiple users' signals that can reduce the measurement requirement to reconstruct every user's signal.

4) Reconstruction Overhead: In Fig. 8 the CS reconstruction overhead of different schemes are compared. The reconstruction overhead for each user in the simulation is first evaluated by the actual average time used by a user to run the reconstruction in Matlab. The overhead is then normalized by being divided by the reconstruction overhead of one user in scheme "conv1". We can see that among all the schemes, our proposed joint recovery scheme can achieve the least overhead. Compared to "conv2", which is the sequential detection scheme with CS reconstruction every sensing period T_p, the overhead for CS reconstruction in our proposed scheme is about 80% lower.

VIII. Conclusion

This paper presents an integrated framework to efficiently perform wideband detection and wideband sensing. Compressed sensing (CS) technique is incorporated with sequential detection to ensure low overhead and more accurate wideband detection. To further identify the subbands occupied by PU, we take advantage of the joint sparsity among neighboring users to achieve effective cooperative wideband sensing. Simulation results demonstrate the significant advantages of our design in reducing the detection delay, increasing the detection accuracy, as well as reducing CS recovery overhead and compressive measurement requirements.

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