Competitive Market with Renewable Power Producers Achieves Asymptotic Social Efficiency

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Abstract—A price-making two-settlement power market in which both conventional generators and renewable power producers (RPPs) participate is studied. It is proved that the Nash Equilibrium (NE) of the market converges to the social optimum as the number of RPPs increases. As a result, social efficiency is asymptotically achieved with a simple market mechanism for integrating RPPs, without the need for an independent system operator (ISO) to perform a centralized stochastic optimization. The analytical derivation of the NE offers an elegant characterization of the market power of the competitive RPPs. The market outcomes predicted by the developed theoretical results are demonstrated by simulation studies.

I. INTRODUCTION

Power systems around the world have recently been experiencing a significant growth of integrated renewable energies such as wind and solar power. What mechanism power system operation should employ to integrate renewable energies (feed-in tariff as one example) has been under active ongoing debates [1]. Considering that an independent system operator (ISO) takes an extended responsibility of economic dispatch (ED), now in the presence of uncertain renewable generation, many works have studied ED approaches based on stochastic optimization and control given probabilistic information of the renewables [2], [3], [4]. Extensive evaluation of the impact of renewable energy integration on the operation cost of power systems and locational marginal prices (LMPs) have been conducted [5], [6], [7].

A major alternative to treating renewable power generation as uncontrollable negative loads is to let renewable power producers (RPPs) participate in power markets, similarly to what conventional generators do. Strategic behaviors of a single RPP in multi-settlement power markets have been analyzed with price-taking assumptions [8] as well as in price-making environments [9], [10], [11], for which stochastic optimization approaches have been explored. On analyzing the behaviors of many RPPs, aggregation of RPPs has been studied with price-taking assumptions in two-settlement markets [12], [13], [14]. In this context, Nash equilibrium (NE) among the aggregating RPPs under several payoff allocation mechanisms has been studied [15], [16], [17], [18], [19]. With a slightly stylized price-making assumption in the day-ahead (DA) market and a fixed real-time (RT) penalty, competition and coalition behaviors of RPPs have been analyzed [20].

In this paper, we study participation of many RPPs in general price-making DA and RT two-settlement power markets. We study a simple mechanism in which each RPP submits a firm power commitment in the DA market, and, by participating in the RT market, is fully responsible for any deviation from it. We provide a closed-form characterization of the NE among all the RPPs in this market. We prove that, as the number of RPPs increases, the NE of the market converges to the social optimum as if an omniscient ISO performs a centralized minimization of the overall expected system cost. The analytical derivation of the NE also offers an elegant characterization of the market power of the competitive RPPs. Simulation studies demonstrate the market outcomes predicted by the developed theoretical results.

The remainder of the paper is organized as follows. Section II establishes the system model of the price-making two settlement market with competitive RPPs. Section III derives the social optimum achieved by an omniscient ISO. Section IV analyzes the market equilibrium with competitive RPPs. Section V offers simulation results that corroborate the derived theoretical results. Section VI concludes the paper.

II. SYSTEM MODEL

A. A Price-Making Two-Settlement Power Market

We consider a two-settlement power market consisting of a day-ahead (DA) market and a real-time (RT) market, and price making (as opposed to price taking) participants in both DA and RT markets. We consider the presence of both conventional generators and renewable power producers (RPPs): the power outputs of the conventional generators are fully controllable, whereas that of the RPPs are not controllable (except for curtailing which will be discussed later), but depend on external factors such as weather. As a result, in the DA market, the power generation of the RPPs at the (future) delivery time are modeled as random variables. Furthermore, we consider that conventional generators are categorized into DA “slow-ramping” ones and RT “fast-ramping” ones: the slow-ramping generators (which are typically cheaper) are to be dispatched in the DA market, and the fast-ramping ones in the RT market.

The general steps of the two-settlement market mechanism that we consider in this paper are summarized as follows:

1) In the DA market,
   a) The DA conventional generators submit their bidding curves to the ISO.
   b) The RPPs submit firm commitments for their power delivery at the future time of the RT market.
Scenario 3
Scenario 2
Scenario 1

2) In the RT market,
   a) The RT conventional generators submit their bid-
      ding curves to the ISO.
   b) The RPPs’ actual generation are realized.
   c) The ISO computes the remaining difference be-
      tween the total generation and load, and performs
      an optimal dispatch of the RT conventional gen-
      erators to resolve the difference.

Before specifying the details of the above steps, we list the
assumptions made in this paper as follows:
1) Transmission network constraints are not considered.
   This is equivalent to considering all the generators, RPPs
   and loads located at a single node.
2) The conventional generators submit their generation cost
   functions truthfully. In other words, we do not consider
   market power issues among the conventional generators.
3) The RPPs have zero variable cost in their generation.

With the above Assumption 1) and 2), it is convenient
 to consider that the market has just a single equivalent
aggregate DA conventional generator and a single equivalent
aggregate RT conventional generator for the ISO to dispatch.
In addition, we consider \( N \) RPPs in the system. Our focus
is to understand the strategic behaviors of the RPPs and the
ensuing consequences on the social welfare. The notations
of the relevant variables are defined as follows:

\[
q^D_A, q^R_T \quad \text{Power dispatch of the (aggregate)}
\]

\[
C^D_G(\cdot), C^R_G(\cdot) \quad \text{Cost functions of the (aggregate)}
\]

\[
L \quad \text{Total (inelastic) load.}
\]

\[
c_i \quad \text{Firm power commitment submitted by RPP} \ i.
\]

\[
c_N \quad \text{The quantity equal to} \ \sum_{i=1}^{N} c_i.
\]

\[
X_i, x_i \quad \text{The random variable modeling the power}
\]

\[
X_N, x_N \quad \text{The random variable modeling the total}
\]

\[
p^D, p^R \quad \text{DA, RT market clearing prices.}
\]

\[
\mathcal{P}_i, \pi_i \quad \text{The realized and expected profit of RPP} \ i.
\]

We now specify the details of how the ISO and the RPPs
interact in the two-settlement market mechanism.

**DA and RT market clearing:** First, we consider that every
RPP takes responsibility of its RT deviation from its DA
firm power commitment. In particular, when performing DA
dispatch of the (aggregate) conventional generator, the ISO
takes the RPPs’ commitments as firm ones, and does not worry
about any possible RT deviations from them. As a result, the
ISO’s DA dispatch is simply given by

\[
q^D_G = L - c_N. \tag{1}
\]

In the RT market, the only sources of a possible difference
between the total generation and load are the deviations of the
RPP’s realized generation from their DA commitments. The
ISO then dispatch the RT conventional generator to resolve
the difference. The ISO’s RT dispatch is thus given by

\[
q^R_T = c_N - x_N. \tag{2}
\]

With (1) and (2), the DA and RT market clearing prices
are the marginal costs for producing one more unit of power
using the DA and RT conventional generators, respectively:

\[
p^D = \left. \frac{dC^D_G(q)}{dq} \right|_{q^D_A}, \quad p^R = \left. \frac{dC^R_G(q)}{dq} \right|_{q^R_N}. \tag{3}
\]

Clearly, the DA and RT market clearing prices depend on the
RPPs’ decisions on \( \{c_i\} \) and their realized generation \( \{x_i\} \),
resulting in a price making two-settlement market.

**Remark 1:** In general, there are three possible shapes of
\( C^R_G(\cdot) \), as depicted in Fig. 1: a) In Scenario 1, RPPs can
sell their excess power in the RT market for profits; b) In
Scenario 2, RPPs cannot sell excess power for profits, but can
curtail them at no cost; c) In Scenario 3, RPPs are penalized
for having excess power, and thus both positive and negative
imbalances are penalized. We note that the results of this paper
are applicable to all possible scenarios.

**RPPs’ profits:** Last but not least, as the RPPs take responsi-
ibility of their own deviations from their DA commitments,
they pay (or possibly get paid) for the dispatch of the RT
cost function \( c_N \) (cf. (2)) in full. Specifically, the profit
earned by an RPP \ i in this two-settlement market is given by

\[
\mathcal{P}_i = p^D(c_N) \cdot c_i - p^R(c_N - x_N) \cdot (c_i - x_i), \tag{4}
\]

where the DA and RT prices are functions of \( c_N \) and \( c_N - x_N \),
respectively.

**B. Benchmark: The Social Optimization Problem**

We now turn to a fundamental question that precedes the
analysis of the two-settlement market, that is, what is the
socially optimal two-stage dispatch? Specifically, in the ideal
case where the ISO has the information and control of all
the players (i.e., DA and RT conventional generators, and
RPPs), we are interested in the DA and RT dispatch that
minimizes the expected overall generation cost to meet the

![Fig. 1. Three scenarios of the RT generation cost function \( C^R_G(\cdot) \).
](image)
load. Understanding the social optimum not only is important in its own right, but also provides a benchmark against which the performance of the market mechanism discussed in the previous sub-section can be compared.

In the following sections, we will first analyze the social optimum of the two-stage dispatch problem, and then analyze the social efficiency of the competitive DA-RT market in which RPPs take responsibility of their own deviations from DA commitments.

III. SOCIAL OPTIMUM FOR TWO-STAGE DISPATCH

In solving the social optimization problem, the ISO is assumed to know all the information about the conventional generators and the RPPs, including the joint probability distribution of the RPPs’ generation \( \{X_i\} \). The goal of the ISO is to minimize the expected overall cost of the system:

\[
\min_{q_G^{DA}} C_{G}^{DA} (q_G^{DA}) + \mathbb{E}_{X_N} \left[ C_{G}^{RT} \left( L - q_G^{DA} - x_N \right) \right],
\]

where \( x_N = \sum_{i=1}^{N} x_i \). Note that, the only free decision variable for the ISO is the DA conventional generation dispatch \( q_G^{DA} \). The RT dispatch must always satisfy \( q_G^{RT} = L - q_G^{DA} - x_N \) to meet the load.

Now, even though there is no commitment from the RPPs when studying the social optimum, we still can define an auxiliary variable \( c_N = L - q_G^{DA} \). This is for the convenience of comparison later with the results in Section IV. Note that this definition is consistent with (1). With a change of variable with the so-defined \( c_N \), the social optimization problem (5) is equivalent to the following,

\[
\min_{c_N} C_{G}^{DA} (L - c_N) + \mathbb{E}_{X_N} \left[ C_{G}^{RT} \left( c_N - x_N \right) \right].
\]

Furthermore, even though there is no market when studying the social optimum, we still can also define the DA and RT prices as follows, again using the change of variable with \( c_N \):

\[
p^f = -\frac{dC_{G}^{DA} (L - c_N)}{dc_N}, \quad p^r = \frac{dC_{G}^{RT} (c_N - x_N)}{dc_N}
\]

Note that these are consistent with the prices in (3).

We now have the following lemma on characterizing the social optimum.

**Lemma 1:** The optimal solution of (6), denoted by \( c_N^* \), is computed from the following condition,

\[
\frac{dC_{G}^{DA} (L - c_N)}{dc_N} \bigg|_{c_N = c_N^*} \mathbb{E}_{X_N}[p^r] + \frac{d\mathbb{E}_{X_N} \left[ C_{G}^{RT} \left( c_N - x_N \right) \right]}{dc_N} \bigg|_{c_N = c_N^*} = 0,
\]

\[
\Leftrightarrow p^f = \mathbb{E}_{X_N}[p^r].
\]

The proof of Lemma 1 follows directly from the optimality condition of (6).

**Remark 2:** An instructive interpretation of Lemma 1 is as follows. Consider an ISO deciding the total commitment \( c_N \) on behalf of the RPPs: The socially optimal total commitment \( c_N^* \) equals the DA market clearing price and the expected RT market clearing price.

While the social optimum can be achieved using Lemma 1, it however requires the ISO to a) know key probabilistic forecast information from the RPPs, and b) performs centralized optimization and control. Instead, in practice, it is very appealing to use market mechanisms to integrate the RPPs into the power system as discussed in Section II-A, which is the focus of the next section.

IV. COMPETITIVE MARKET WITH RENEWABLE POWER PRODUCERS

In this section, we analyze the two-settlement market mechanism as described in Section II-A, where each RPP submits a DA firm power commitment, and takes responsibility for any RT deviation from it. From the RT realized profit of RPP \( i \), its DA expected profit is given by

\[
\pi_i = \mathbb{E}_{X_N}[p^r] (c_N^* - c_i^*) - c_i^* \mathbb{E}_{X_N} \left[ p^r (c_N^* - x_N) \right] + \mathbb{E}_{X_N} \left[ p^r (c_N^* - x_N) \right] \cdot x_i
\]

When participating in the two-settlement market, each RPP has total freedom in choosing its DA commitment \( c_i \), and thus a strategic RPP would like to choose one that maximizes its expected profit \( \pi_i \) (cf. (10)). The strategic behaviors of the RPPs can thus be studied in a non-cooperative game theoretic framework as in the reminder of the section.

A. Nash Equilibrium Achieves Asymptotic Social Efficiency

We study the following non-cooperative game modeling the strategic behaviors of the RPPs in the two-settlement market, which we term the commitment game:

1) **Players:** the set of RPPs participating in the DA-RT market: \( \mathcal{N} = \{1, \ldots, N\} \).

2) **Strategies:** the firm power commitments made by the RPPs. \( \{c_i\} \).

3) **Payoffs:** Each RPP \( i \)'s payoff is its expected profit (10).

We now state the main result of this paper.

**Theorem 1:** Social efficiency is achieved at each pure Nash equilibrium (NE) of the commitment game as \( N \rightarrow \infty \).

**Proof:** Suppose the strategy profile \( \{c_i^{ne}, \ldots, c_N^{ne}\} \) is a pure NE of the commitment game, and \( c_N^{ne} = \sum_{i=1}^{N} c_i^{ne} \) is the total commitment of the RPPs at this pure NE. Since each RPP’s expected profit is maximized at this pure NE, \( \{c_i^{ne}\} \) must satisfy the following necessary best response conditions:

\[
\frac{d\pi_i}{dc_i} \bigg|_{(c_1, \ldots, c_N) = (c_1^{ne}, \ldots, c_N^{ne})} = 0, \quad \forall i \in \mathcal{N}.
\]

Summing up the \( N \) equations above, we have

\[
\sum_{i \in \mathcal{N}} \frac{d\pi_i}{dc_i} \bigg|_{(c_1, \ldots, c_N) = (c_1^{ne}, \ldots, c_N^{ne})} = 0.
\]
With some algebra, (12) simplifies to the following condition:

\[(N - 1) \cdot \left( p^f \left( c_{N}^{*,ne} - E_{X_N} \left[ p^f \left( c_{N}^{*,ne} - x_N \right) \right] \right) + \frac{dE_{X_N}[P_N]}{dc_{N}^{*,ne}} \right) = 0, (13)\]

where \( P_N \equiv p^f c_N - p^f (c_N - x_N) \cdot (c_N - x_N) \), which is the total profit of the RPPs. As a result, when \( N > 1 \),

\[p^f \left( c_{N}^{*,ne} - E_{X_N} \left[ p^f \left( c_{N}^{*,ne} - x_N \right) \right] \right) = \frac{\frac{dE_{X_N}[P_N]}{dc_{N}^{*,ne}} |_{c_{N}^{*,ne}}}{N - 1}, (14)\]

and the right hand side of (14) converges to zero as \( N \to \infty \) (under mild technical conditions). Therefore, as the number of RPPs goes to infinity, the social efficiency condition (9) is achieved, meaning that the dispatch of the DA and RT conventional generators achieves the minimum overall cost in the system.

Remark 3 (Social Efficiency-RPPs’ Profit Tradeoff): The result from Theorem 1, in particular (13), has a very interesting and intuitive interpretation. As the number of RPPs vary, the NE of the market makes a tradeoff between achieving social efficiency (by equalling the DA price and the expected RT price) and maximizing the RPPs’ total expected profit (by setting \( \frac{dE_{X_N}[P_N]}{dc_{N}^{*,ne}} |_{c_{N}^{*,ne}} = 0 \)). For the case of only one RPP, i.e., \( N = 1 \), the NE maximizes the expected profit of that RPP. As the number of RPPs increases, the NE of the market moves from generating the maximum expected total profit for the RPPs to the social optimum. As \( N \to \infty \), \( (p^f - E_{X_N} \left[ p^f \right]) \to 0 \), and social efficiency is achieved asymptotically (cf. (9)).

\[N \to \infty, \mbox{RPPs do not have market power at all, and the market becomes socially efficient.}\]

V. SIMULATION

In this section, we conduct simulation studies to demonstrate the main results in the previous sections. We use quadratic functions to model the (aggregate) cost functions of the conventional generators in the DA and RT markets as follows:

\[C_G^{DA} (q) = \frac{1}{2} \alpha_{G}^{DA} \cdot q^2 + \beta_{G}^{DA} \cdot q, \]

\[C_G^{RT} (q) = \frac{1}{2} \alpha_{G}^{RT} \cdot q^2 + \beta_{G}^{RT} \cdot q. \]

Accordingly, with \( q_{G}^{DA} = L - c_N \) and \( q_{G}^{RT} = c_N - x_N \) (cf. (1), (2)), the DA and RT market prices become (cf. (3) and (7))

\[p^f = -\frac{dC_G^{DA}}{dc_{N}} = \alpha_G^{DA} \cdot (L - c_N) + \beta_G^{DA}, \]

\[p^f = \frac{dC_G^{RT}}{dc_{N}} = \alpha_G^{RT} \cdot (c_N - x_N) + \beta_G^{RT}. \]

The socially optimal total commitment of the RPPs \( c_N^{*,ne} \) (cf. (8)) and the total commitment at the NE of the market \( c_N \) (cf. (13)) have closed form expressions as follows:

\[c_N^{*,ne} = \frac{\frac{\alpha_G^{DA} L + \alpha_G^{RT} \mu_N + \beta_G^{DA} - \beta_G^{RT}}{\alpha_G^{DA} + \alpha_G^{RT}}}{N + 1} \cdot \mu_N, \]

\[c_N = \frac{\frac{N \cdot (\alpha_G^{DA} L + \beta_G^{DA}) + (N + 1) \cdot \alpha_G^{RT} \mu_N}{N + 1} \cdot \mu_N}{N + 1}. \]

To measure the social welfare, the expected overall system cost is calculated as \( C_G^{DA} + E_{X_N} \left[ C_G^{RT} \right] \).

We employ the following parameters for the simulations.

<table>
<thead>
<tr>
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<th>α_G ($/\text{MW}^2$)</th>
<th>β_G ($/\text{MW}$)</th>
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</thead>
<tbody>
<tr>
<td>DA</td>
<td>0.01</td>
<td>15</td>
</tr>
<tr>
<td>RT</td>
<td>0.02</td>
<td>30</td>
</tr>
</tbody>
</table>

We simulate with all the RPPs’ generation being independent and identically distributed (IID) Gaussian random variables, \( X_i \sim N (\mu, \sigma^2), \forall i \in N \). Consequently, \( X_N = \sum_{i \in N} X_i \sim N (\mu_N, \sigma_N^2) \), where \( \mu_N = N \cdot \mu \), and \( \sigma_N = \sqrt{N} \cdot \sigma \). Throughout all the simulations, we consider a fixed (aggregate) expectation and (aggregate) standard deviation of the RPPs’ total generation \( X_N \), with \( \mu_N = 500 \text{MW} \), and \( \sigma_N = 30 \text{MW} \). The simulated mean and variance of each individual RPP would thus depend on the number of RPPs \( N \). For example, if there are \( N = 100 \) RPPs, then each of them would have \( \mu_i = \frac{\mu_N}{N} = 5 \text{MW} \) and \( \sigma_i = \frac{\sigma_N}{\sqrt{N}} = 3 \text{MW} \).

The total load is set to be 1000MW.

In our simulations, we vary the number of RPPs \( N \), and evaluate the social optimum and the NE of the two-settlement market for the above setting. The expected overall system costs are plotted in Figure 2. Because the probability distribution of the total generation of the RPPs are kept fixed (\( \mu_N = 500 \text{MW} \)
and $\sigma_N = 30\lambda [W]$, the social optimum stays fixed. As the number of RPPs $N$ increases, it is observed that the expected overall system cost at the market NE converges quickly to the social optimum. The DA market clearing prices and the expected RT market clearing prices are plotted in Figure 3. When $N$ is small, there is a clear discrepancy between the DA and the expected RT prices. As $N$ increases, the two prices converge to each other. Lastly, we plot the expected total profit of all the RPPs in Figure 4. Clearly, as the number of RPPs increases, competition among themselves becomes greater, and their total expected profit decreases.

VI. CONCLUSION

We study a simple mechanism that integrates RPPs in a price-making DA-RT two-settlement power market: each RPP submits a firm DA power commitment, and, by participating in the RT market, is responsible for any RT deviation from it. It is proved that, the NE among the RPPs in the market converges to the social optimum as the number of RPPs increases. Thus, competition among the RPPs promotes the social welfare. The analytical derivation of the NE offers an elegant characterization of the market power of the RPPs. The developed theoretical results are demonstrated by simulation studies.

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