# An Incentive Compatible Market Mechanism for Integrating Demand Response into Power Systems

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***Abstract*—In this paper, we propose a market mechanism that allows demand response providers (DRPs) to participate in a two-settlement electricity market as power suppliers alongside conventional generators. Each DRP bids her demand reduction capacity and cost rate, and the independent system operator (ISO) schedules the power dispatch to minimize the overall system cost. We show that, with the proposed mechanism, truthful bidding by the DRPs is achieved at a Nash equilibrium (NE), and as a result, the social welfare is maximized. The theoretical results are corroborated by simulation studies.**

## I. Introduction

Demand Response (DR) plays an important role in sustaining an efficient and reliable power system, especially in the presence of a high level penetration of renewable energies. In the current implementations of DR, an electricity consumer is encouraged to shift or reduce her demand in situations where the reliability of the power system is compromised.

There are two typical types of DR programs: price-based and incentive-based. In the price-based programs, pricing schemes such as Real-Time Pricing (RTP) [1][2], Time-of-Use (TOU) Pricing [3][4], and Critical Peak Pricing (CPP) [5] are employed to encourage users to adjust their electricity usage. In the incentive-based programs, in comparison, users receive extra monetary payments as incentives to reduce their demand. A typical example of such a program is baseline-based: a user's baseline energy consumption when no DR is performed is established, and rewards are issued based on the difference between the baseline and their actual reduced consumption. While such baselines are typically estimated by the load serving entities (LSEs), a number of works have studied mechanisms in which users submit their baselines to the LSE/system operator. [6] discusses a price-taking market between consumers and an aggregator, and designs a DR contract based on the probability of call which is incentive compatible on reduction capacity and asymptotically incentive compatible on baseline. [7] focuses on consumer-aggregator interaction also in a price-taking market setting, where the probability of a DR event is small: A Vickrey–Clarke–Groves (VCG)-inspired mechanism for self-reporting baseline and marginal utility is proposed. A more recent work [8] studies a price-making market consisting of DRs and a system operator (SO): A truthful mechanism that only elicits baseline is proposed, and the SO's cost is almost optimal under a similar assumption of a very small DR event probability. [9] proposed a DR market structure where the DR target is determined in the wholesale market, and a supplemental DR market is introduced for DR bidding. A reversed uniform price auction is performed to elicit customers' true preferences. However the market is not analyzed from a social welfare perspective. Another line of related work studied supply function bidding, where each DR customer's preference is represented by a supply function and is submitted to the market [10][11]. These works primarily focus on the setting of allocating a certain amount of demand reduction among the DRPs. Last but not least, interesting DR experiments in the real world include [12] for end-consumers, among others.

In this paper, we consider a set of DRPs participating in a two-settlement, day-ahead (DA) and real-time (RT), market. We propose a simple bidding mechanism where each DRP submits her DR capacity and cost rate, (rather than the baselines which we assume are estimated by the system operator,) and the independent system operator (ISO) schedules them accordingly alongside the conventional generators to minimize the total system cost. We prove that truthful bidding by the DRPs results in a Nash equilibrium (NE), and hence achieves the social optimum. Here, our model does not yet consider the uncertainties of DRPs at DA (e.g., a DRP at DA may not be sure about her DR capacity at RT in the next day [13]). Nevertheless, the proposed mechanism in the two-settlement market structure is designed so that extending its use in the presence of uncertainties is straightforward in a spirit similar to [14], [15]. As such, the results for the deterministic case here lay the groundwork for incorporating uncertainties of DRPs which is left for future work.

The rest of the paper is organized as follows. Section II describes the proposed market mechanism and formulates the problems faced by the DRPs and the ISO. Section III analyzes the outcome of the mechanism from a game theoretic perspective. Section IV provides simulation results that support the theoretical results. Section V concludes the paper.

## II. Problem Formulation

### A. System Model

We base our mechanism for integrating DRPs in a DA-RT two-settlement power market, in which both the conventional generators and the DRPs participate as energy suppliers. Note that although the DRPs are not actually generating power on

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TABLE. I. NOTATIONS

| | |
|---|---|
| $\alpha_i^T$ | True cost rate of DRP$_i$ |
| $\alpha_G^D$, $\alpha_G^R$ | Cost coefficients of the DA & RT conventional generators |
| $C_i(\cdot)$ | Cost of DRP$_i$ for providing DR service |
| $L$ | Total load |
| $c_i^T$ | DRP$_i$'s true DR capacity |
| $c_i^D$, $c_i^R$ | DA & RT capacities submitted by DRP$_i$ |
| $\alpha_i$ | Cost rate submitted by DRP$_i$ at DA (unchanged at RT) |
| $d_i^D$, $d_i^R$ | DA commitment and RT dispatch for DRP$_i$ |
| $q_G^D$, $q_G^R$ | Power dispatch for conventional generators in the DA & RT markets |
| $p^D$, $p^R$ | DA & RT market clearing prices |
| $\mathcal{P}_i^D, \mathcal{P}_i^R, \mathcal{P}_i$ | The payoff earned by DRP$_i$ in the DA & RT markets, and in total |

their own, they curtail their demand in the same amount as they "generate" power, at cost rates equal to their marginal utilities. We consider two sets of conventional generators participating in the DA and RT market, respectively. For example, those that are slow-ramping are in the DA market, whereas the fast-ramping ones are in the RT market. Moreover, We consider the market to be *price-making*: the participants, specifically the DRPs, have the power to influence market prices by adjusting their bids in the market. An overview of the proposed market mechanism is as follows:

1) In the DA market
   - The DRPs submit their DA information to the ISO based on their estimates of the cost rates and capacities;
   - The DA generators submit their information to the ISO;
   - Based on these DA information, the ISO optimally schedules the energy resources to meet the total load, and pays the energy resources accordingly.
2) In the RT market
   - The DRPs submit their RT information to the ISO based on their realized cost rates and capacities;
   - The RT generators submit their information to the ISO;
   - Based on these RT information, the ISO performs another optimal scheduling to balance the generation and load, and pays/charges the energy resources accordingly.

We make the following model assumptions in this paper:

1) Transmission network constraints are not considered, meaning that we consider a single node where all the DRPs, DA and RT conventional generators are located.
2) The DA and RT conventional generators submit their information truthfully. In other words, we do not analyze the strategic behavior of conventional generators, but focus on that of the DRPs. We note that Assumptions 1) and 2) allow us to replace all the DA generators with one equivalent DA generator and all the RT generators with one equivalent RT generator.
3) The DA and RT conventional generators have linear cost functions. and the RT generator is no cheaper than the DA generator, i.e. $\alpha_G^D \leq \alpha_G^R$ (cf. Table I). This assumption is intuitive as the RT generator has faster ramp rates.
4) We assume that the ISO has an accurate estimate of the total load, and the DRPs' total capacity bids in the DA market is no greater than the total load. This is intuitive as the DRPs' total capacity of demand reduction is typically much lower than their actual total demand.
5) To model demand response as a more economic resource than conventional generators, we assume that the DRPs are less expensive than the conventional generators, i.e.,

$$\alpha_i^T < \alpha_G^D \ \left(\leq \alpha_G^R\right), \forall i \in \mathcal{N}. \tag{1}$$

We now provide the model details for the DRPs and the conventional generators.

**DR providers' model**: We consider a set of DRPs $\mathcal{N} = \{1, 2, \cdots, N\}$. Each DRP$_i$ has a linear cost rate for providing one unit of demand reduction, denoted as $\alpha_i^T$, and also a maximum demand reduction capacity, denoted as $c_i^T$. The superscript $T$ stands for the *true* values of these parameters which are private information to the DRP herself. Moreover, we assume a deterministic model where each DRP$_i$ has accurate knowledge of $\alpha_i^T$ and $c_i^T$ at DA. In other words, we do not model the uncertainties of these parameters at DA, and relaxing this assumption is left for future work. As such, DRP$_i$'s cost function is

$$C_i(d) = \begin{cases} \alpha_i^T \cdot d & 0 \leq d \leq c_i^T \\ \infty & d \geq c_i^T \end{cases} \tag{2}$$

Where $d$ is the amount of demand reduction. The infinity term suggests that DRP$_i$ is unable to deliver any demand response higher than her true capacity.

**Conventional generators' model**: From the above assumptions, we consider a DA generator and a RT generator with linear cost functions and coefficients $\alpha_G^D \leq \alpha_G^R$. We assume each of these two has sufficient generation capacity to support the total load on his own. As such, the cost functions of the DA and RT generators are $C_G^D(q) = \alpha_G^D \cdot q,\ \ C_G^R(q) = \alpha_G^R \cdot q$, in which $q \geq 0$ is the amount of energy dispatched in the DA or RT market from the corresponding DA or RT generator.

### *B. The Proposed Mechanism and Market Clearing Process*

We now describe the proposed mechanism of market participation and clearing. The timeline of the actions of the market participants in this mechanism is summarized in Figure 1.

**DA market:** At DA, our mechanism requires the following:

- Each DRP$_i$ submits a bid consisting of her cost rate, denoted by $\alpha_i$, and the day-ahead bid of her DR capacity, denoted by $c_i^D$, to the ISO. Notably, these bids need not be truthful.
- The DA generator submits his true cost rate $\alpha_G^D$ to the ISO.
- DA market clearing: Upon receiving these bids, the ISO performs an optimal power dispatch *treating all bids as truthful*:

$$\min_{\{0 \leq d_i^D \leq c_i^D\},\ q_G^D \geq 0} \alpha_G^D \cdot q_G^D + \sum_{i=1}^{N} \alpha_i \cdot d_i^D \tag{3}$$

$$s.t. \qquad q_G^D + \sum_{i=1}^{N} d_i^D = L. \tag{4}$$

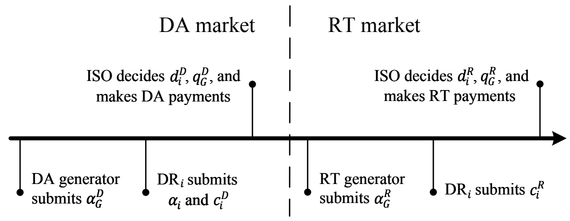


Fig. 1. Timeline of the proposed DR mechanism.

Specifically, solving (3) is equivalent to a) sorting the submitted cost rates of all the DRPs and the DA generator in ascending order, and b) scheduling these energy resources from the cheapest to the most expensive, until the market is cleared, i.e., the total load is met. Assumptions 4) and 5) in the preceding subsection implies that every DRP will be scheduled for her submitted capacity bid. In other words, the ISO will use up all the more economic (and scarce) demand reduction first, before using the conventional generators. As such, the amount of demand reduction the ISO schedules at DA for DRP$_i$ is the same as her submitted capacity,

$$d_i^D = c_i^D, \tag{5}$$

which we term as DRP$_i$'s DA *commitment*. The reason of this terminology is that this amount may be different from the actual dispatch of DRP$_i$ at RT, which will be determined in the RT market as detailed later. The rest of the load serving is provided by the DA generator, whose power dispatch is

$$q_G^D = L - \sum_{i \in \mathcal{N}} d_i^D. \tag{6}$$

Lastly, the DA *market clearing price* is set as the marginal cost of producing one more unit of power using the next available power resource, which in this case is the marginal cost of the DA generator (cf. Assumption 4):

$$p^D = \alpha_G^D. \tag{7}$$

- Next, the ISO issues a payment to each DRP$_i$ as

$$\mathcal{P}_i^D = p^D \cdot d_i^D = \alpha_G^D \cdot c_i^D, \tag{8}$$

and its payment to the DA generator is:

$$\mathcal{P}_G^D = p^D \cdot q_G^D = \alpha_G^D \cdot \big(L - \sum_{i=1}^{N} c_i^D\big). \tag{9}$$

**RT market:** At RT, our mechanism requires the following:

- Each DRP$_i$ submits a bid of her real-time DR capacity $c_i^R$ to the ISO. Again, the bid needs not be truthful.
- The RT generator submits his true cost rate $\alpha_G^R$ to the ISO.
- RT market clearing: Upon receiving these bids, the ISO performs another *optimal* power dispatch *treating all bids as truthful*. Importantly, the ISO allows *rescheduling of the DRPs*, meaning that the DRPs are rescheduled in the RT market *regardless of their scheduled DA commitment*. In other words, while a) the DA generator's dispatch scheduled in the DA market is physically binding due to its slow ramping and inflexibility to change at RT, b) the DRPs' DA commitments are not physically binding and can be changed at RT due to their flexibility. As such, equivalently, the ISO solves the optimal RT dispatch such that the total supply from the DA generator, the RT generator and the DRPs meets the total demand $L$:

$$\min_{\{0 \le d_i^R \le c_i^R\},\ q_G^R \ge 0} \alpha_G^R \cdot q_G^R + \sum_{i=1}^{N} \alpha_i \cdot d_i^R \tag{10}$$

$$s.t. \qquad q_G^D + q_G^R + \sum_{i=1}^{N} d_i^R = L, \tag{11}$$

where $d_i^R$ and $q_G^R$ are the RT dispatch of DRP$_i$ and the RT generator, respectively. From (11) and (6) we have

$$q_G^R + \sum_{i=1}^{N} d_i^R = \sum_{i=1}^{N} d_i^D. \tag{12}$$

As such, the ISO is equivalently dispatching the power resources from the DRPs and the RT generator to meet the DRPs' total DA commitment $\sum_{i=1}^{N} d_i^D$.
Similarly to DA, the RT *market clearing price* $p^R$ is set as the marginal cost of producing one more unit of power using the next available power resource (which can be either the RT generator or one of the DRPs, as demonstrated in the proof of Theorem 2).
- The RT payoff from the ISO to DRP$_i$ is:

$$\mathcal{P}_i^R = p^R \cdot \big(d_i^R - d_i^D\big) = p^R \cdot \big(d_i^R - c_i^D\big). \tag{13}$$

In other words, *each DRP$_i$ is charged/paid for her deviation from her DA commitment scheduled by the ISO*. We note that this is a key property for the mechanism to be effective. The ISO also makes a payment to the RT generator:

$$\mathcal{P}_G^R = p^R \cdot q_G^R = p^R \cdot \Big(\sum_{j=1}^{N} d_j^D - \sum_{j=1}^{N} d_j^R\Big). \tag{14}$$

We note that, in the above mechanism, the DRPs bid capacities in both the DA and RT markets, but cost rates in the DA market only. Allowing the DRPs to bid cost rates in the RT market as well is left for future work.

**Remark 1.** While the DRPs need not bid truthfully, if the bids result in an ISO's RT dispatch of DRP$_i$ that exceeds her true capacity, which can only happen when $c_i^R > c_i^T$, she will not be able to deliver the scheduled dispatch. If this case happens, we impose a sufficiently large penalty for DRP$_i$. Consequently, every DRP will be mindful and not submit bids that would lead to a RT dispatch that exceeds her capacity.

### C. DR Providers' Profits

The overall profit earned by DRP$_i$ in the DA-RT market is the sum of her DA and RT payoff in (8) and (13), minus her cost in (2) for providing DR service:

$$\begin{aligned}\mathcal{P}_i &= \mathcal{P}_i^D + \mathcal{P}_i^R - C_i(d_i^R) \\ &= d_i^D \cdot p^D + p^R \cdot (d_i^R - d_i^D) - \alpha_i^T \cdot d_i^R\end{aligned} \tag{15}$$

*D. DR Providers' Strategic Bidding in a Non-Cooperative Game*

We consider each DRP strategically bidding in the DA and RT markets to maximize her overall profit. We analyze their strategic behaviors in a game theoretic framework. Formally, we define the following non-cooperative game:

1) Players: a set of DRPs $\mathcal{N} = \{1, 2, \cdots, N\}$.
2) Strategies: Each DRP's strategy is a tuple consisting of her bids of cost rate, DA capacity and RT capacity. A strategy profile of the RPPs is a set of $N$ tuples $\{(\alpha_i, c_i^D, c_i^R), \forall i \in \mathcal{N}\}$.
3) Payoffs: DRPs' profit $\mathcal{P}_i, \forall i \in \mathcal{N}$, as defined in (15).

We define the following solution concept of the game:

**Definition 1.** A pure Nash equilibrium (NE) is defined as a set $\left\{(\alpha_i^\star, c_i^{D,\star}, c_i^{R,\star}), \forall i \in \mathcal{N}\right\}$, such that for $\forall i \in \mathcal{N}$,

$$(\alpha_i^\star, c_i^{D,\star}, c_i^{R,\star}) \in \underset{(\alpha_i, c_i^D, c_i^R)}{\operatorname{argmax}} \mathcal{P}_i\Big((\alpha_i, c_i^D, c_i^R), (\alpha_{-i}^\star, c_{-i}^{D,\star}, c_{-i}^{R,\star})\Big),$$

where $(\alpha_{-i}^\star, c_{-i}^{D,\star}, c_{-i}^{R,\star})$ represents the set of other DRPs' bids at a NE except DRP$_i$.

At a NE, no DRP has an incentive to deviate from her own strategy in pursuit of a higher payoff. In such a sense we say that a NE produces a *stable* market outcome.

## III. Main Result

We now provide the main theorems of this paper.

**Theorem 1.** (*Incentive Compatibility*) Under the proposed mechanism detailed in Section II, *truthful* bidding is a Nash equilibrium, *i.e.*, $(\alpha_i^\star, c_i^{D,\star}, c_i^{R,\star}) = (\alpha_i^T, c_i^T, c_i^T), \forall i \in \mathcal{N}$.

The strategy we employ to prove Theorem 1 is to prove for a general sub-problem: We show that, regardless of what the DPRs bid for their cost rates, the mechanism will always induce truthful bidding of their capacities. Formally, we have the following theorem:

**Theorem 2.** Under the proposed mechanism detailed in Section II, given any cost-rate bids, truthful bidding of DA and RT capacities is a Nash equilibrium for the DRPs.

*Proof of Theorem 2:* W.l.o.g., we focus on an arbitrary DRP$_i$. We assume that all DR providers other than DRP$_i$ are submitting their true DR capacities in both DA and RT markets, *i.e.*, $(c_j^D, c_j^R) = (c_j^T, c_j^T), \ \forall j \neq i$. The bids for cost rates, $\{\alpha_j < \alpha_G^D, \forall j \in \mathcal{N}\}$, can be arbitrary.

We first compute the truth telling case as a baseline for our analysis. Let $(c_i^D, c_i^R) = (c_i^T, c_i^T)$. Following the mechanism, $d_i^D = d_i^R = c_i^T$, $p^D = \alpha_G^D$, $p^R = \alpha_G^R$, and DRP$_i$'s profit is

$$\mathcal{P}_i^T = c_i^T \cdot (\alpha_G^D - \alpha_i^T). \tag{16}$$

We now analyze different cases of untruthful bidding strategies for DRP$_i$, and show that the total profit of DRP$_i$ is maximized if she follows the truthful bidding strategy.

***Case 1:*** *Truthful bidding in DA market, Untruthful bidding in RT market* $\left(c_i^D = c_i^T\right)$: In the DA market we have $d_i^D = c_i^D = c_i^T$ and $p^D = \alpha_G^D$. To analyze the RT market, we consider the following two sub-cases:

1.1) *Underbidding in RT market* $\left(c_i^R < c_i^T\right)$: DRP$_i$ underbids $\gamma$ unit of energy, *i.e.*, $c_i^R = c_i^T - \gamma$, where $\gamma > 0$. In this case, the RT dispatch for player $i$ is $d_i^R = c_i^R = c_i^T - \gamma$, and the ISO faces a shortfall. No other DRP can provide any additional DR service as they are truth telling in both DA and RT markets. Hence the RT generator would have to be dispatched to cover this shortfall. The price in the RT market is $p^R = \alpha_G^R$. The profit of player $i$ is:

$$\begin{aligned}\mathcal{P}_i &= p^D \cdot d_i^D + p^R \cdot (d_i^R - d_i^D) - \alpha_i^T \cdot d_i^R \\ &= \alpha_G^D \cdot c_i^T - \alpha_G^R \cdot \gamma - \alpha_i^T \cdot (c_i^T - \gamma) \\ &= \underbrace{(\alpha_G^D - \alpha_i^T) \cdot c_i^T}_{\mathcal{P}_i^T} - \underbrace{(\alpha_G^R - \alpha_i^T) \cdot \gamma}_{>0} < \mathcal{P}_i^T \end{aligned} \tag{17}$$

1.2) *Overbidding in RT market:* Player $i$ overbids in the RT market, *i.e.*, $c_i^R > c_i^T$. In this case the RT market will have a surplus in DR capacity. As a result the RT market price will decrease compared to the truth-telling case, *i.e.* $p^R < \alpha_G^R$. We note that two scenarios can be considered here. The first scenario is that DRP$_i$ is the most expensive DRP (according to their cost rate bids), *i.e.* $i = \operatorname{argmax}_{j\in\mathcal{N}} \alpha_j$. In this case overbidding does not result in over-scheduling, and $d_i^R = d_i^D (= c_i^T)$. The profit of DRP$_i$ remains unchanged. The second scenario is that DRP$_i$ is *not* the most expensive DRP, *i.e.* $i \neq \operatorname{argmax}_{j\in\mathcal{N}} \alpha_j$. In this case, DRP$_i$ is over-scheduled in the RT market and some other more expensive DRP(s) are under-scheduled. However, such RT dispatch cannot be physically delivered by DRP$_i$, meaning that she will make sure this scenario does not occur (cf. Remark 1).

***Case 2:*** *Overbidding in the DA market* $\left(c_i^D > c_i^T\right)$: Suppose player $i$ overbids $\delta$ unit of energy in the DA market, $\delta > 0$. We have that $d_i^D = c_i^D = c_i^T + \delta$, and $p^D = \alpha_G^D$ (cf. (7)). We consider three sub-cases for DRP$_i$ in the RT market:

2.1) *Underbidding in the RT market* $\left(c_i^R < c_i^T\right)$: Suppose DRP$_i$ underbids $\gamma$ units in the RT market, *i.e.*, $c_i^R = c_i^T - \gamma$, $\gamma > 0$. In this case there is a shortfall in the RT market, and $d_i^R = c_i^R = c_i^T - \gamma$, $p^R = \alpha_G^R$. As such,

$$\begin{aligned}\mathcal{P}_i &= \alpha_G^D \cdot (c_i^T + \delta) - \alpha_G^R \cdot (\gamma + \delta) - \alpha_i^T \cdot (c_i^T - \gamma) \\ &= (\alpha_G^D - \alpha_i^T) \cdot c_i^T - \underbrace{(\alpha_G^R - \alpha_G^D) \cdot \delta}_{>0} - \underbrace{(\alpha_G^R - \alpha_i^T) \cdot \gamma}_{>0} \\ &< \mathcal{P}_i^T \end{aligned} \tag{18}$$

2.2) *Overbidding in the RT market but less than DA capacity bid* $\left(c_i^T < c_i^R < c_i^D\right)$: Suppose player $i$ overbids $\gamma$ units of energy. We have that $c_i^R = c_i^T + \gamma$, $\delta > \gamma > 0$. In this case the shortfall still exists in the RT market, and $d_i^R = c_i^T + \gamma$, which means DRP$_i$ would be unable to deliver it in real time. As such, she will make sure this scenario does not occur (cf. Remark 1).

2.3) *Overbidding in RT market to more than DA capacity bid* $\left(c_i^T < c_i^D \leq c_i^R\right)$: similarly to the previous case, we can write $c_i^R = c_i^T + \gamma$, $\gamma \geq \delta > 0$. In this scenario, it is always the case that $d_i^R \in [c_i^T + \delta,\ c_i^T + \gamma]$, again resulting in DRP$_i$ unable to deliver in real time. As such, she will make sure this scenario does not occur (cf. Remark 1).

***Case 3:*** *Underbidding in the DA market* $\left(c_i^D < c_i^T\right)$: Consider the case where DRP$_i$ underbids $\delta$ units of energy in the DA market, $\delta > 0$. For this case we have $d_i^D = c_i^D = c_i^T - \delta$, and $p^D = \alpha_G^D$. Similarly to the previous case, we consider the following scenarios for the RT market:

3.1) *The RT capacity bid is no higher than the DA capacity bid* $\left(c_i^R \leq c_i^D < c_i^T\right)$: In this case, $d_i^R = c_i^R$, and the RT market price is $p^R = \alpha_G^R$. By underbidding from her DA report, player $i$ is sacrificing much of her RT payoff in exchange of little utility gain. Specifically,

$$\begin{aligned} \mathcal{P}_i &= p^D \cdot d_i^D + p^R \cdot (d_i^R - d_i^D) - \alpha_i^T \cdot d_i^R \\ &= \alpha_G^D \cdot c_i^D + \alpha_G^R \cdot (c_i^R - c_i^D) - \alpha_i^T \cdot c_i^R \\ &= (\alpha_G^D - \alpha_G^R) \cdot c_i^D + (\alpha_G^R - \alpha_i^T) \cdot c_i^R \\ &\leq (\alpha_G^D - \alpha_G^R) \cdot c_i^D + (\alpha_G^R - \alpha_i^T) \cdot c_i^D \\ &= (\alpha_G^D - \alpha_i^T) \cdot c_i^D < \mathcal{P}_i^T \end{aligned} \tag{19}$$

3.2) *The RT capacity bid is higher than the DA capacity bid* $\left(c_i^R > c_i^D\right)$: This is the most interesting case which corresponds to withholding capacity at DA and releasing it at RT. Because DRP$_i$ increases her RT capacity bid from her DA capacity bid, the ISO in the RT market would a) accept some (to none) of her extra capacity before turning to other players (if any) with higher submitted cost rates, and b) does not dispatch the RT generator. From a), it is always true that DRP$_i$'s actual RT dispatch satisfies $d_i^R \geq c_i^T - \delta$, where the equality represents the special case where DRP$_i$ has the highest submitted cost rate. From b), we know that the RT price must be the submitted marginal cost of one of the DRPs, e.g., $\alpha_k$. We have that $p^R = \alpha_k < \alpha_G^D \leq \alpha_G^R$. We also have the capacity constraint that $d_i^R \leq c_i^T$ (cf. Remark 1). The payoff for player $i$ is:

$$\begin{aligned} \mathcal{P}_i &= p^D \cdot d_i^D + p^R \cdot (d_i^R - d_i^D) - \alpha_i^T \cdot d_i^R \\ &= (\alpha_G^D - \alpha_k) \cdot (c_i^T - \delta) + (\alpha_k - \alpha_i^T) \cdot d_i^R \end{aligned} \tag{20}$$

Here the relationship between $\alpha_k$ and $\alpha_i^T$ depends on the submitted cost rates. Therefore we consider the following three sub-scenarios:

a) $\alpha_k > \alpha_i^T$. Because $d_i^R \leq c_i^T$, we have that

$$\begin{aligned} \mathcal{P}_i &\leq (\alpha_G^D - \alpha_k) \cdot (c_i^T - \delta) + (\alpha_k - \alpha_i^T) \cdot c_i^T \\ &= (\alpha_G^D - \alpha_i^T) \cdot c_i^T - (\alpha_G^D - \alpha_k) \cdot \delta < \mathcal{P}_i^T. \end{aligned} \tag{21}$$

b) $\alpha_k = \alpha_i^T$. We have that

$$\mathcal{P}_i = (\alpha_G^D - \alpha_i^T) \cdot (c_i^T - \delta) < \mathcal{P}_i^T. \tag{22}$$

c) $\alpha_k < \alpha_i^T$. Because $d_i^R \geq c_i^T - \delta$, we have that:

$$\begin{aligned} \mathcal{P}_i &\leq (\alpha_G^D - \alpha_k) \cdot (c_i^T - \delta) + (\alpha_k - \alpha_i^T) \cdot (c_i^T - \delta) \\ &= (\alpha_G^D - \alpha_i^T) \cdot (c_i^T - \delta) < \mathcal{P}_i^T. \end{aligned} \tag{23}$$

All the cases above together show that truthful capacity-bidding in both DA and RT market is optimal for DRP$_i$ when all the other DRPs are truthful-bidding. ■

**Remark 2.** We note that Theorem 1 is a special case of Theorem 2, and Theorem 2 immediately implies Theorem 1. This is because, *regardless of DRPs' cost rate bids*, Theorem 2 implies that the profit for each DRP$_i$ is always (16). As such, truth-telling of the cost rates is a straightforward NE.

**Remark 3** (*Individual Rationality*)**.** From (16) and (1), at NE, every player receives a non-negative payoff, which suggests that she will be willing to sign up for the DR program. Individual rationality is a desired property for a market mechanism as it guarantees an incentive for participation.

**Remark 4** (*Social Efficiency*)**.** The property of truthful bidding immediately implies that social efficiency can be achieved, because the ISO schedules the energy resources based on the bids to minimize the overall social cost.

**Remark 5** (*Insensitivity to DRPs' Cost Rate Estimation*)**.** One practical issue when integrating DRPs in a system is that, unlike conventional generators, the cost rate of a DRP is sometimes not even clearly defined due to human factors, let alone being estimated accurately. Nonetheless, we show in Theorem 2 that an accurate estimate of a DRP's cost rate is not essential in the sense that it does not affect the market outcome. The only assumption we have on DRPs' cost rates is Assumption 5), which is a reasonable one. We note that we will need to relax this assumption when considering the uncertainty of DRPs, which is left for future work.

## IV. Simulation

In this section, we conduct simulation studies to demonstrate the main results of this paper. The case study we use for this purpose consists of four DRPs as detailed in Table II. The

TABLE. II. PARAMETERS OF THE DR PROVIDERS

| DRP #1 | | DRP #2 | | DRP #3 | | DRP #4 | |
|---|---|---|---|---|---|---|---|
| $\alpha_1^T$ $(\frac{\$}{KWh})$ | $c_1^T$ $(KW)$ | $\alpha_2^T$ | $c_2^T$ | $\alpha_3^T$ | $c_3^T$ | $\alpha_4^T$ | $c_4^T$ |
| 8 | 30 | 7 | 20 | 3 | 40 | 9 | 10 |

total load is $150KW$ and the cost coefficient of the DA and RT generators are $\alpha_G^D = 15$ $(\frac{\$}{KWh})$ and $\alpha_G^R = 20$ $(\frac{\$}{KWh})$. We perform two simulation studies in this section. The first simulation is to verify the *truthfulness* property of the proposed DR mechanism. The second simulation demonstrates the benefits of using the proposed DR mechanism by the ISO in minimizing the total system cost.

- **Simulation 1:** Assuming that DRP #2, #3, and #4 are truth-telling in both the DA and RT markets, we let DRP #1 change her capacity bids in both markets. The total payoff of DRP #1 w.r.t. different DA & RT capacity bids is shown in Figure 2. We note that, as we showed in the proof of Theorem 2, the cost-rate bids has less strategic value to the DRPs than the capacity bids. As such, in this simulation we focus on the capacity bids in the DA and RT markets, and

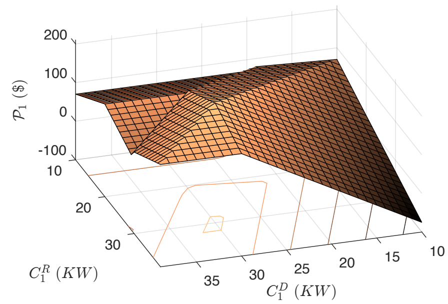


Fig. 2. Payoff of $\mathrm{DRP}_1$ w.r.t. its capacity bids in DA & RT markets.

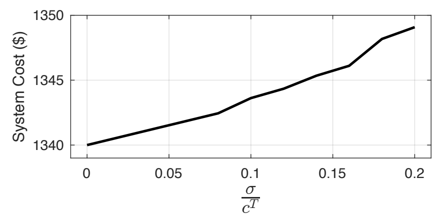


Fig. 3. Expected system cost achieved by the ISO solving a stochastic optimization problem, as its uncertainty about the DRPs increases.

assume that DRP #1 submits her cost-rate bid truthfully. It is clear that the payoff of DRP #1 is maximized when $c_1^D = c_1^R = c_1^T = 30$, verifying the truth-telling property.

- **Simulation 2:** As an alternative to the proposed mechanism, we consider the case where the ISO does not elicit the capacities from the DRPs but relies on its own estimation of them. Specifically, the ISO assumes some probability distribution functions for the capacities of the DRPs, and solves a stochastic optimization based on these PDFs to schedule the DRPs in the DA market and minimize the total expected cost. To be clear, we model the DRPs' capacities as deterministic without uncertainties, and the use of probability distributions here is solely to model that *the ISO itself does not know the capacities of DRPs with certainty*.

  We assume that the ISO uses a multivariate Gaussian PDF for the DRPs' capacities, with a) the mean being the true capacities, and b) the covariance matrix being a diagonal matrix where the ratio of the standard deviation to the mean, $\frac{\sigma_i}{c_i^T}$, is the same for all the DRPs. Here we focus on the effect of the ISO's uncertainty in the DRP capacities, and assume the ISO knows the DRPs' cost rates accurately. We plot the ISO's optimized system cost as a function of the ratio $\frac{\sigma_i}{c_i^T}$ in Figure 3. Clearly, the minimum system cost is achieved when $\frac{\sigma_i}{c_i^T} = 0$, i.e., when the ISO accurately knows the DRPs' capacities. As the ratio $\frac{\sigma_i}{c_i^T}$ increases, the system cost also increases as a result of the higher uncertainty in the ISO's knowledge about the DRPs' capacities. In comparison, in the proposed mechanism, the ISO is able to elicit the DRPs' true capacities from the DRPs themselves as opposed to making its own forecast. We note that, in practice, it is a lot more difficult for the ISO to estimate the DRPs' capacities than for the DRPs themselves, and the proposed mechanism resolves this issue by the truth-telling property (cf. Theorem 1).

## V. Conclusion

A simple mechanism for integrating demand response providers into a price-making two-settlement power market is proposed. Each DRP has a limited DR capacity and a cost rate of providing DR, and participates in the proposed market mechanism alongside the conventional generators. Each DRP submits bids of DR capacity and cost rate to maximize her payoff, while the ISO schedules the power resources to minimize the overall social cost. The strategic behavior among the DRPs is studied in a non-cooperative game, and it is proved that truth-telling is an NE. The proposed mechanism also guarantees individual rationality and social efficiency. The theoretical results are corroborated by simulation studies. As this paper assumes deterministic models for DR capacities and cost rates, market outcomes in the presence of DRPs' uncertainties are left for future work.

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