Dynamic Joint Outage Identification and State Estimation in Power Systems

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***Abstract*—Joint outage identification and state estimation is studied in power systems in which cascading outages dynamically develop and network states dynamically evolve. A recursive algorithm is developed that computes in closed form the joint posterior of cascades and network states at every time step. A beam search technique is employed that prevents the number of cascades to compute from growing exponentially. Because the joint posterior is a sufficient statistic for jointly identifying the cascades and estimating the states, the derived closed forms can be applied to develop the optimal dynamic joint detector and estimator under any performance criterion. We simulate cascading line outages with uncertain network states in the IEEE 14-bus and 57-bus systems, and the proposed algorithm is evaluated for dynamically identifying outages and estimating states at every time step. It is observed that retaining just a few cascades in the beam search can achieve a joint identification and estimation performance close to that with all cascades retained.**

I. Introduction

Lack of situational awareness in power transmission systems has been a prominent cause of blackouts [1]. In particular, earlier component outages (e.g., tripping of lines and generators), if not attended in time, can quickly escalate to large-scale cascading failures that render major parts of the system out of power. When component failures dynamically develop, it is crucial to identify and keep track of in real time which components are in outage so that informed control actions can be taken accordingly. A closely related problem is estimating the states (conventionally defined as voltage phasors) in the power network [2]. In the presence of cascading failures, state estimation becomes much more challenging than under normal conditions, as it needs to be performed with outage identification simultaneously.

Outage identification has been primarily studied in a static setting. Methods based on exhaustive search over the set of outage hypotheses have been developed to detect single and two-line outages in power transmission networks [3], [4]. A recent work exploiting the sparsity of line outages has been developed to identify more-than-two-line outages [5]. Another recent work developed message passing algorithms to identify an arbitrary number of line-outages [6]. One common assumption in these works is that the network states are known. In practice, however, states are not known accurately, and must be estimated by using the same set of measurements based on which outage identification is performed. Early work on performing state estimation when there are topological errors include [7]–[9], among others. A key idea therein is to relax the binary status of lines (connected vs. disconnected) to real numbers, so that heuristics on estimating continuous variables can be used to estimate the line status together with the states. Recently, sparsity of topological errors has been exploited to further improve state estimation with such topological error processing [10]. In these works, the primary goal is to improve state estimation performance despite topological errors. Optimal joint outage identification and state estimation has been developed in [11]. There, a Bayesian framework is employed, and the joint posterior of the outages and states is computed in closed form.

We study joint outage identification and state estimation in a dynamic setting. Previous works that focus on dynamic state estimation include [12]–[15], among others. In this paper, in addition to dynamically evolving states, we consider dynamically developing outages that are typical in cascading failures. We model the outage dynamics as follows: at every time step, one additional line outage occurs. To optimally track the outage patterns and estimate the states simultaneously, we develop a recursive algorithm that efficiently computes the joint posterior (given all the past sensor measurements) of the cascade history and the current states. This joint posterior then enables us to perform joint outage identification and state estimation optimally under any performance criterion. As the cascade failures develop, the number of possible cascades grows exponentially with time. To address this problem, we employ a beam search technique that keeps a limited number of most probable cascades at every time step. We simulate cascading failures in the IEEE 14-bus and 57-bus systems, and apply the developed recursive algorithm to jointly identify outages and estimate states at every time step. We observe that the joint identification and estimation performance with just a few cascades retained in the beam search is close to that with all cascades retained.

The remainder of the paper is organized as follows. The system model is established in Section II, and the problem

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of joint dynamic outage identification and state estimation is formulated. A recursive algorithm that computes in closed form the joint posterior of cascades and network states is developed in Section III. The algorithm is evaluated with simulated cascading failures in Section IV. Conclusions are drawn in Section V.

II. Problem Formulation

We study joint outage identification and state estimation in a power transmission network that dynamically evolves over time. We begin with the following general form of network state and observation equations:

$$x_{t+1} = f_t(x_t) + u_t, \quad (1)$$
$$z_t = h_t(x_t) + v_t, \quad (2)$$

where the quantities are defined as follows.

- The network states at time t are denoted by x_t. The state transition from time t to $t+1$ is driven by $f_t(\cdot)$ and u_t.
- The measurements at time t from the sensors that monitor the grid are denoted by z_t. $h_t(\cdot)$ and v_t are the observation function and noise at time t.

In power systems, x_t is conventionally defined to include the voltage magnitudes and phase angles at all the buses. z_t can be measurements of voltage and current magnitudes and phase angles, power flows, and power injections. The observation function $h_t(\cdot)$ that relates z_t to x_t is determined by

- power network topology and parameters,
- sensor types and locations, and
- power flow equations.

Under normal operating conditions, network topology does not change, network parameters change much more slowly than state dynamics, and sensors are static. As a result, $h_t(\cdot)$ stays approximately the same over time. Knowing $h_t(\cdot)$, the primary task is then estimating x_t from z_t. However, when component failures and outages occur, the network topology changes, and so does $h_t(\cdot)$. As these outages (and hence $h_t(\cdot)$) are not known to the operator a-priori, they must be identified in conjunction with estimating x_t.

A. Outage Dynamic Model

We focus on identifying *cascading failures of transmission lines* while simultaneously estimating network states. In particular, we study the following model by which outages evolve:

- At time 0, the network is in normal condition.
- For every $t \geq 0$, from time t to $t+1$, one additional line outage occurs.

We denote the line outage pattern at time t by $\mathcal{H}_t$. An illustration of cascading line outages is depicted in Figure 1.

We assume that the outage transition probability $p(\mathcal{H}_{t+1}|\mathcal{H}_0^t, z_1^t)$ is known, where $\mathcal{H}_0^t \triangleq \{\mathcal{H}_0, \ldots, \mathcal{H}_t\}$, and $z_1^t \triangleq \{z_1, \ldots, z_t\}$. In other words, given a particular cascade history $\mathcal{H}_0^t$ and all the past observations z_1^t, we can compute the probability for the next failure to occur at each of the remaining lines. For example, one can compute estimates of the states $\hat{x}_t$ based on $\mathcal{H}_0^t$ and z_1^t, and hence that of the power flows on all the transmission lines. Then, based on how close each line's power flow is to the corresponding line flow capacity, the probability for the next failure to occur at each line can be estimated.

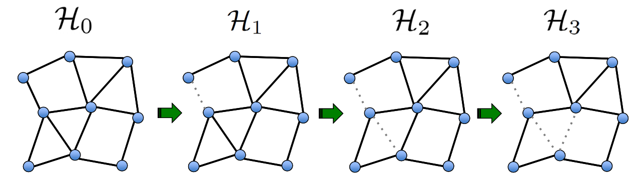

Fig. 1. The development of cascading failures over time. The solid and dashed lines denote the connected and disconnected transmission lines, respectively.

We further assume that, once the outage pattern $\mathcal{H}_t$ is given, $f_t(\cdot)$ and $h_t(\cdot)$ are determined.

B. State Dynamic Model

Because of the nonlinearity of the power flow equations, $h_t(\cdot)$ is in general a nonlinear observation function (cf. Chapter 2 of [2]). With a crude estimate of the states, a linearized model can be obtained using the Jacobian of $h_t(\cdot)$. In this paper, we employ the following linear dynamics in modeling state transitions and observations:

$$x_{t+1} = F_t x_t + u_t, \quad (3)$$
$$z_t = H_t x_t + v_t, \quad (4)$$

where u_t and v_t are independent zero-mean sequences. We note that such a linear dynamic model has been previously employed in forecasting-aided state estimation [14]. With renewable energy sources (e.g., wind and solar) integrated into power systems, the temporal characteristics of renewable power generation can also be modeled into the state transition matrix F_t. We further assume that x_0, u_t and v_t follow Gaussian distributions.

C. Optimal Joint Outage Identification and State Estimation

Given the outage and state dynamic models, our goal is to perform optimal joint identification of $\mathcal{H}_0^t$ and estimation of x_t at each time t. For this, our approach is to compute the *joint posterior* of the cascade history and the state vector,

$$p(\mathcal{H}_0^t, x_t|z_1^t) = p(x_t|\mathcal{H}_0^t, z_1^t)p(\mathcal{H}_0^t|z_1^t). \quad (5)$$

We note that the joint posterior is a sufficient statistic for jointly identifying the cascades and estimating the states. Thus, knowing this joint posterior, we can make the joint identification and estimation decision optimally under any performance criterion.

III. Recursive computation of joint posterior

In this section, we provide an algorithm that computes the joint posterior (5) recursively in closed form. This algorithm consists of three building blocks: a) computing the posterior conditional probability density function (PDF) of the states $p(x_t|\mathcal{H}_0^t, z_1^t)$, b) computing the posterior probability mass function (PMF) of the cascade history $p(\mathcal{H}_0^t|z_1^t)$, and c) a beam search technique that keeps the number of cascades manageable.

A. Posterior Conditional PDF of States

First, given the cascade history $\mathcal{H}_0^t$, all the state transition matrices $F_1, \ldots, F_t$ and observation matrices $H_1, \ldots, H_t$ are known (cf. (3) and (4)). As a result, $p(x_t|\mathcal{H}_0^t, z_1^t)$ is a Gaussian PDF, whose mean and covariance matrix can be tracked via a Kalman filter [16]. Similarly, prediction and smoothing, namely computing $p(x_{t+1}|\mathcal{H}_0^t, z_1^t)$ and $p(x_0^{t-1}|\mathcal{H}_0^t, z_1^t)$, can also be performed using Kalman filters. Here, for later use in computing the posterior PMF of cascade history, we present the recursive formulas for computing $p(x_{t+1}|\mathcal{H}_0^t, z_1^t) \sim N(\hat{x}_{t+1|t,\mathcal{H}_0^t}, \Sigma_{t+1|t,\mathcal{H}_0^t})$ as follows:

$$\hat{x}_{t+1|t,\mathcal{H}_0^t} = F_t \hat{x}_{t|t-1,\mathcal{H}_0^{t-1}} + F_t K_t (z_t - H_t \hat{x}_{t|t-1,\mathcal{H}_0^{t-1}}), \tag{6}$$

$$\Sigma_{t+1|t,\mathcal{H}_0^t} = F_t (I - K_t H_t) \Sigma_{t|t-1,\mathcal{H}_0^{t-1}} F_t^T + Q_t, \tag{7}$$

with the gain matrix

$$K_t = \Sigma_{t|t-1,\mathcal{H}_0^{t-1}} H_t^T (H_t \Sigma_{t|t-1,\mathcal{H}_0^{t-1}} H_t^T + R_t)^{-1}, \tag{8}$$

where $R_t = \mathrm{Cov}(v_t)$, $Q_t = \mathrm{Cov}(u_t)$, and I is the identity matrix. Recursive formulas for $p(x_0^t|\mathcal{H}_0^t, z_1^t)$ can be derived similarly [16].

B. Posterior PMF of Cascade History

To compute $p(\mathcal{H}_0^t|z_1^t)$ recursively, we first apply Bayes' formula as follows:

$$p(\mathcal{H}_0^t|z_1^t) = \frac{p(z_t|\mathcal{H}_0^t, z_1^{t-1}) p(\mathcal{H}_0^t|z_1^{t-1})}{p(z_t|z_1^{t-1})}. \tag{9}$$

We note that

$$p(\mathcal{H}_0^t|z_1^{t-1}) = p(\mathcal{H}_t|\mathcal{H}_0^{t-1}, z_1^{t-1}) p(\mathcal{H}_0^{t-1}|z_1^{t-1}), \tag{10}$$

where the outage transition probability $p(\mathcal{H}_t|\mathcal{H}_0^{t-1}, z_1^{t-1})$ is known by assumption. Next, from the linear observation model (4) and the fact that $p(x_t|\mathcal{H}_0^{t-1}, z_1^{t-1})$ is a Gaussian PDF, it follows that $p(z_t|\mathcal{H}_0^t, z_1^{t-1})$ is also a Gaussian PDF. Thus, it suffices to compute its mean and covariance matrix. Employing the recursively-computed mean and covariance matrix of x_t given z_1^{t-1} and $\mathcal{H}_0^{t-1}$, after some simple algebra, we have

$$\zeta_{t|t-1,\mathcal{H}_0^t} \triangleq \mathbb{E}(z_t|\mathcal{H}_0^t, z_1^{t-1}) = H_t \hat{x}_{t|t-1,\mathcal{H}_0^{t-1}}, \tag{11}$$

$$C_{t|t-1,\mathcal{H}_0^t} \triangleq \mathrm{Cov}(z_t|\mathcal{H}_0^t, z_1^{t-1}) = H_t \Sigma_{t|t-1,\mathcal{H}_0^{t-1}} H_t^T + R_t. \tag{12}$$

Accordingly, we have the following lemma:

Lemma 1. *The posterior PMF of the cascade history can be computed by*

$$p(\mathcal{H}_0^t|z_1^t) = \frac{p(\mathcal{H}_t|\mathcal{H}_0^{t-1}, z_1^{t-1}) p(\mathcal{H}_0^{t-1}|z_1^{t-1})}{f(z_1^t) \det(C_{t|t-1,\mathcal{H}_0^t})^{1/2}} \cdot \exp\left(-\frac{1}{2}\|z_t - \zeta_{t|t-1,\mathcal{H}_0^t}\|^2_{C^{-1}_{t|t-1,\mathcal{H}_0^t}}\right), \tag{13}$$

where $f(z_1^t)$ is a normalization factor such that $\sum_{\mathcal{H}_0^t} p(\mathcal{H}_0^t|z_1^t) = 1$, and the notation $\|x\|^2_\Sigma$ denotes $x^T \Sigma x$ for any positive definite matrix Σ.

Recursive Joint Posterior Algorithm

Initialization:
Let the set of cascades at time 0, $\{\mathcal{H}_0\}$, contain the healthy grid.
Let $\hat{x}_{0|-1} = \mathbb{E}(x_0)$, and $\Sigma_{0|-1} = \mathrm{Cov}(x_0)$.
At time t ($t \geq 1$),
Generate possible cascades up to time t:
Based on each cascade retained up to time $t-1$, $\mathcal{H}_0^{t-1}$, generate all cascades $\mathcal{H}_0^t$ that add one additional line outage at time t. Collect all cascades so generated in the set $\{\mathcal{H}_0^t\}$.
Compute the joint posterior of cascades and states recursively:
For each cascade collected in $\{\mathcal{H}_0^t\}$, compute $p(\mathcal{H}_0^t|z_1^t)$ from (13), $\hat{x}_{t+1|t,\mathcal{H}_0^t}$ from (6), and $\Sigma_{t+1|t,\mathcal{H}_0^t}$ from (7).
Keep no more than K cascades up to time t using beam search:
Among all the collected cascades $\{\mathcal{H}_0^t\}$, keep up to K of them with the highest non-zero $p(\mathcal{H}_0^t|z_1^t)$, discard the remaining cascades, and normalize the retained $p(\mathcal{H}_0^t|z_1^t)$ to sum up to 1.

Clearly, based on $\hat{x}_{t|t-1,\mathcal{H}_0^{t-1}}$, $\Sigma_{t|t-1,\mathcal{H}_0^{t-1}}$ and $p(\mathcal{H}_0^{t-1}|z_1^{t-1})$, we can now compute $\hat{x}_{t+1|t,\mathcal{H}_0^t}$, $\Sigma_{t+1|t,\mathcal{H}_0^t}$ and $p(\mathcal{H}_0^t|z_1^t)$ from (6),(7) and (13).

C. Beam Search

A remaining challenge of tracking the cascade history is that the size of the set of all possible cascades grows exponentially with time (cf. Figure 1). Specifically, the number of possible cascades up to time t is of the order $\frac{L!}{(L-t)!}$, where L is the number of transmission lines in the network. Thus, it is not computationally efficient to keep track of all possible $\mathcal{H}_0^t$ and compute all $p(\mathcal{H}_0^t|z_1^t)$. To address this, we employ a beam search technique that keeps the *most likely* K cascades at each time step [17], where K is a fixed number.

For each cascade up to the previous time step, one of the remaining (up to L) lines can fail, leading to a cascade up to the current time step. Using beam search, we need only to compute up to KL joint posteriors $p(x_t|\mathcal{H}_0^t, z_1^t) p(\mathcal{H}_0^t|z_1^t)$ at each time step t. In summary, we have the *Recursive Joint Posterior Algorithm* for recursively tracking the joint posterior of cascades and states.

IV. Simulation Results

We simulate cascading failures of lines in the IEEE 14-bus (cf. Figure 2) and 57-bus systems using the software toolbox MATPOWER [18]. The simulation starts from the baseline topology with no outage, and develops until 5 lines are in outage. At each time step, the next failure occurs uniformly randomly among the remaining lines, and we rule out the cases in which the network becomes disconnected. In our simulations, we let the power injections at all the buses stay static but uncertain, with a diagonal covariance matrix, implying independently (but not identically) distributed power injections. We denote by κ the ratio between the standard deviation and the mean of a power injection, which indicates how accurately we know this power injection. We employ the DC power flow model [19]. Thus, the power injections fully determine all the power flows and bus voltage phase angles given an outage pattern, and hence can be viewed as the network states x_t [11]. Accordingly, we focus on evaluating

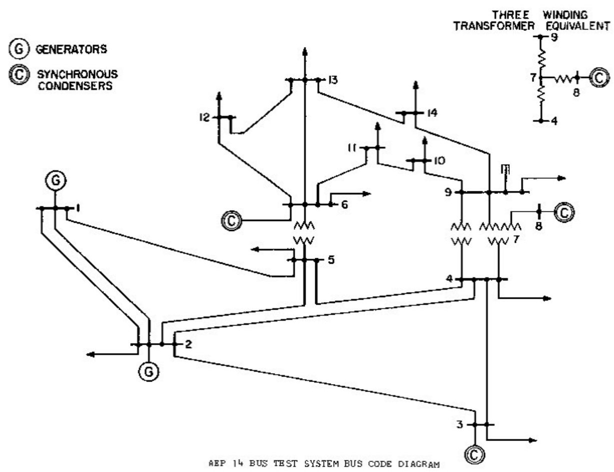

Fig. 2. IEEE 14-bus system.

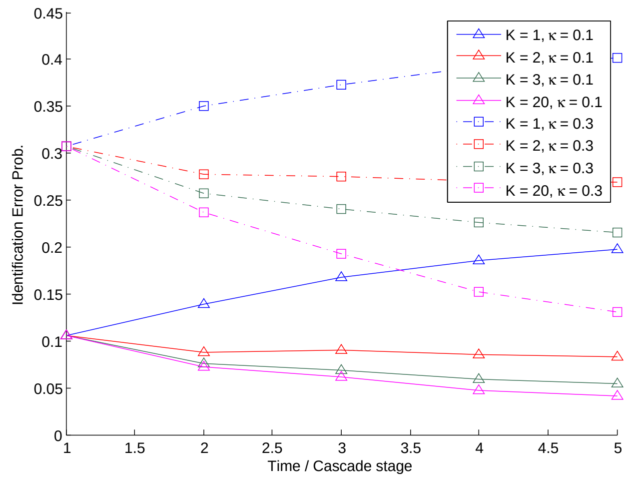

Fig. 3. Error probability for identifying the outage pattern at different cascade stages in the IEEE 14-bus system.

the performance of identifying cascading line outages with the network states being unknown but static. We employ phasor measurement units (PMUs) to measure voltage phase angles at buses, and assume that the PMU measurement noise has zero mean and a standard deviation of 0.3 degree. This degree of accuracy conforms to the IEEE standard for PMUs [20], although we note that recent development has further improved the state-of-the-art PMU accuracy to 0.01 degree [21].

We employ the Recursive Joint Posterior Algorithm for computing the joint posterior of cascades and states in real time as time increases from 1 to 5. At each time step t $(1 \leq t \leq 5)$, we employ the maximum a-posteriori probability (MAP) rule and declare $\hat{\mathcal{H}}_0^t \triangleq \text{argmin}_{\mathcal{H}_0^t} p(\mathcal{H}_0^t|z_1^t)$ as the cascade identification decision. In practice, our primary interest in real time is the *current time*'s outage pattern $\mathcal{H}_t$, instead of the entire cascade history $\mathcal{H}_0^t$. Thus, in evaluating the algorithm, we define that an error occurs at time t if $\hat{\mathcal{H}}_t$ is different from the true outage pattern at this current time. Note that, even if the declared cascade history has errors, it is possible that the current outage pattern implied by this cascade history is still correct. Furthermore, even if an error was made in declaring an outage pattern at a previous time, it is still possible that we correctly declare the true outage pattern at the current time.

In the IEEE 14-bus system, we employ 6 PMUs at buses $1, 3, 7, 11, 12,$ and 14. These PMU locations are selected with a criterion previously developed to optimize the identification of single line outages, and thus provide an optimal performance for correct outage identification at time step 1 in our setting. We refer the reader to [11] for detailed derivations of these optimal locations. As a heuristic, better identification at the initial stage can reduce identification error propagation at later stages in a cascade. We plot the error probabilities for identifying the current outage pattern at times $t = 1, \ldots, 5$ in Figure 3. Each data point is averaged over 10000 Monte Carlo runs. In this figure, we compare the identification performance for two levels of accuracy of state knowledge: $\kappa = 0.1$ and $\kappa = 0.3$. For each level, we compare the performance for $K = 1, 2, 3$ and 20 cascade histories to keep in the beam search. When $K = 1$, we observe that the error probability increases as a cascade develops. This is due to an error propagation phenomenon as follows: since we only keep the most probable cascade history at every time step, if an error occurs in any time step, this error will be kept in the next time step, and the correct cascade history will be lost forever. Interestingly, once we keep two possible cascades in the beam search, the error probability decreases as a cascade develops, meaning that the effect of error propagation is effectively reduced. Another reason for the decreasing error probability as t increases is that the states are static in our simulation, and hence the accuracy of our knowledge on states improves at every time step with the computed posterior on states. This further suggests that accurate outage identification plays a key role in achieving accurate state estimation. We perform a similar experiment in the IEEE 57-bus system, in which we employ 20 PMUs, and consider the case of $\kappa = 0.1$ as the accuracy of the state knowledge. The identification error probabilities are plotted in Figure 4. We observe trends that are similar to those in the IEEE 14-bus system. It is worth noting that the locations of the PMUs can significantly affect the joint cascade identification and state estimation performance. Optimization of sensor locations for tracking cascading failures is left for future work.

A natural question that arises is: how many retained cascades in the beam search are sufficient? To answer this question, we plot in Figure 5 the identification error probabilities as a function of K for different cascade stages (t) in the IEEE 14-bus system. We observe a sharply diminishing return on reducing the error probability as K increases. In particular, for the case of $\kappa = 0.1$, we never observe more than 13 cascades with non-zero posterior probabilities (defined as greater than 10^{-5} in our simulation) at any time. In other words, with $K = 13$, it is essentially guaranteed that we are achieving the same joint outage identification and state

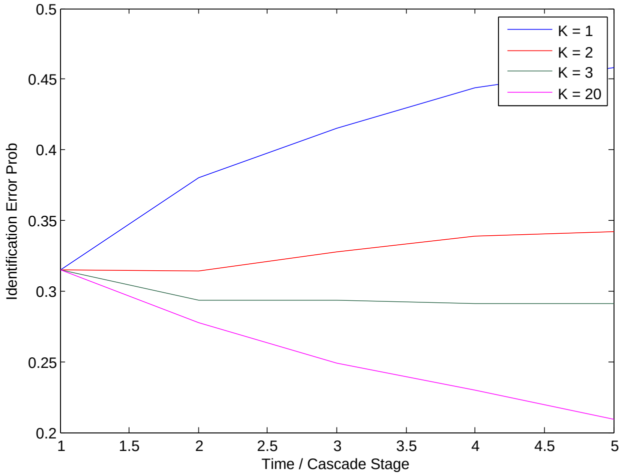

Fig. 4. Error probability for identifying the outage pattern at different cascade stages in the IEEE 57-bus system.

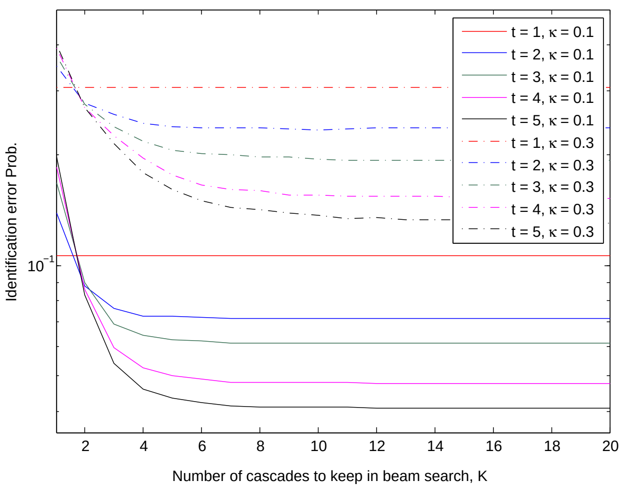

Fig. 5. Error probability for identifying the outage pattern with different numbers of cascades retained in beam search in the IEEE 14-bus system.

estimation performance as if $K = \infty$. A similar phenomenon is observed for the IEEE 57-bus system.

V. Conclusion

We have studied jointly identifying outages and estimating states in a power system in which cascading line outages dynamically develop and states dynamically evolve. We have modeled the outage dynamics to have one additional line failure at every time step, and modeled the states to follow linear dynamics. A recursive algorithm is developed that computes in closed form the joint posterior of the cascade history and the network states given all the past sensor observations. Because the joint posterior is a sufficient statistic for jointly identifying the cascades and estimating the states, based on the developed closed forms, dynamic joint outage identification and state estimation can be optimally performed under any performance criterion. Since the number of possible cascades grows exponentially with time, we have employed a beam search technique that keeps a limited number of cascades when computing the joint posterior. We have evaluated the proposed algorithm for identifying cascading line outages at every time step with uncertain network states in the IEEE 14-bus and 57-bus systems. Our results indicate that keeping just a few cascades in the beam search suffices to successfully control identification error propagation, and to achieve a joint outage identification and state estimation performance close to that with all cascades retained.

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