Interference Strength Alignment and Uplink Channel Allocation in Linear Cellular Networks

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***Abstract*—We find the uplink channel allocation that maximizes the total throughput in linear (one-dimensional) cellular systems, under arbitrary fairness constraints defined by resource distribution functions. We exploit a revenue-cost separation principle in this multi-cell interference channel problem, and equivalently transform maximizing the total throughput into minimizing the total interference cost of all cells. The key idea in obtaining the optimal channel allocation is the alignment of interference signal strengths. We show that with very low complexity, a complete optimal channel allocation can be constructed by rippling along all the cells the proposed interference strength alignment procedure. From analyzing the consequence of the interference strength alignment, the superiority of the optimal channel allocation over CDMA is justified. Numerical results are provided to demonstrate the performance gap between these two schemes.**

I. Introduction

We study the optimal uplink channel allocation (CA) of linear (i.e., one-dimensional) cellular communication networks. The single cell cases take the form of multiple access channels [8], for which the optimal power and channel allocation schemes in Gaussian frequency selective channels are well understood [5], [13]. The multi-cell cases have an added interference channel nature, for which (weighted) throughput maximization by power and channel allocation is a non-convex optimization, and is NP-hard to solve [11], [12]. For general (including cellular) interference networks, much work has been done in reducing the complexity of solving this non-convex problem (although still NP) [4], [14], [15]. With convexified utility functions or other convex approximations, (polynomial time) low complexity methods have also been developed [6], [7], [10].

Recently, progress in interference alignment [2] and the deterministic channel model [1] has provided new perspectives into managing interference. Combining these two leads to the idea of aligning the interference in bit-levels, or signal strengths [3], [15].

In this paper, we exploit the idea of interference signal strength alignment in multi-cell uplink channel allocation. We first separate optimizing fairness and throughput by introducing the resource distribution function (rdf), which can be designed in a natural way to match the fairness requirement. We then assume rdf is given as a fairness constraint on CA, and focus on finding the optimal CA that maximizes the total throughput.

With a revenue-cost separation principle, in our problem setting, throughput maximization is equivalently transformed into interference cost minimization. For every cell, we sort the interference strengths (received at its base station) generated by users in each of its neighboring cells. We then align the i^{th} $(i = 1, 2, \ldots)$ *strongest interference in each of its neighboring cells* to co-exist in the same channel. We prove that once the alignment is done for every cell, the resulting CA minimizes the total interference cost, and hence maximizes the total throughput. For linear cellular networks, we show that alignment of all the cells can be done in a rippling manner. As a result, this procedure of obtaining the complete optimal CA has a low complexity of $O(n_{cell} m \log(m))$, (where n_{cell} is the number of cells, and m is the number of channels.) As will be shown, our methods enable both analytical and numerical comparisons of optimal CA with CDMA schemes.

The rest of the paper is organized as follows. The system model is established in Section II, where several key steps are taken to clarify and simplify the problem structure. In Section III, the complete uplink optimal channel allocation using interference strength alignment is derived. In Section IV, the performance of the optimal CA is compared with typical CDMA schemes through two examples with different fairness requirements. Conclusions are drawn in Section V.

II. System Model

We consider the uplink channel allocation of a linear cellular system with base stations (BS) positioned on a straight line. We make the following assumptions: i) Users within the same cell use orthogonal frequency channels. ii) A frequency reuse factor of 1 is applied among all the cells. iii) Interference is treated as noise, (and hence no interference cancellation or joint decoding is used.) iv) Interference from users not from the immediate neighboring cells is ignored. Thus, each cell has two interfering cells (Figure 1). v) There are m parallel channels with unit bandwidth. In every cell, all the m channels are utilized, (i.e., there is no vacant channel in any cell.) vi) Channels experience frequency flat fading, and the noise spectral density is flat at all receivers. vii) Each user transmits *at its own* uniform power spectral density (PSD) over all the channels it occupies. Accordingly, *we focus on the problem of channel allocation, assuming that users do not vary their selected power levels.*

A. Approximate Rate Function

Employing the Shannon capacity formula for Gaussian channels [8], we have $R = \log(1 + \mathsf{SINR})$ bits/sec/Hz. We approximate this formula in the following way:

$$\log(1 + \mathsf{SINR}) \approx \max(0, \log(\mathsf{SINR})) \quad (1)$$

It can be verified that the maximum deviation from the R.H.S. of (1) to the L.H.S. is 1 bit/sec/Hz, which occurs when $\mathsf{SINR} = 0dB$. Furthermore, note that (1) $= \log(\mathsf{SINR})$ *provided that* $\mathsf{SINR} \geq 0dB$, and this leads to the decomposition of the problem into two steps:

1) Active user selection: in each channel, we select the users that are expected to have $\mathsf{SINR} \geq 0dB$, so that the zero floor becomes redundant. (The inactive users are then dropped from the objective function in this channel.)
2) Optimization with respect to the selected active users, with $R \approx \log(\mathsf{SINR})$.

Note that without $\mathsf{SINR} \geq 0dB$, $\log(\mathsf{SINR})$ can be arbitrarily negative. Thus, choosing the correct active users in step 1) is crucial for the approximation to be close.

B. Revenue-Cost Separation Principle

From now on, we will assume that the active user selection is properly done in each channel so that the zero floor in (1) can be equivalently dropped. In cellular scenarios with in-cell orthogonalization, (i.e. interference only comes from other cells,) this assumption is typically easy to satisfy. Thus, we have the following separation principle for an active user i's rate in any one channel:

$$\begin{aligned} R_i = \log(\mathsf{SINR}_i) &= \log\left(\frac{g_{ii}P_i}{\sum_{k\neq i} g_{ki}P_k + N_i}\right) \\ &= revenue_i - cost_i \end{aligned} \quad (2)$$

where $revenue_i \triangleq \log(g_{ii}P_i), cost_i \triangleq \log(\sum_{k\neq i} g_{ki}P_k + N_i)$, and k runs over all co-channel users of user i. P_k is the transmit PSD of user k, N_i is the noise spectral density at receiver i, and g_{ki} is the channel gain from transmitter k to receiver i. As will be shown in Section III, this revenue-cost separation principle has a significant impact in both analysis and design — it greatly reduces the complexity of finding the optimal CA in multi-cell cases.

C. Resource Distribution Function (rdf)

Definition 1: The *resource distribution function* $rdf(i)$ ($i = 1, \ldots, n$, where n is the total number of users,) is defined to be the *number* of channels allocated to user i, or equivalently, the *amount* of bandwidth allocated to user i.

Note that there are many different CAs that correspond to the same rdf, which may result in very different total throughput due to the differences in multi-cell co-channel user selections. With the concept of rdf, the channel allocation problem is decomposed into two steps: i) decide rdf, and ii) decide CA given rdf. For step i), the design of rdf should match users' demands and channel conditions: users with higher target rates and/or worse channel conditions should have more resources (channels) distributed to them. In this paper, we assume that step i) has been properly done, i.e. rdf is given according to the fairness requirement. (Different cells can have different rdf.) We then investigate the optimal channel assignment in all cells that maximizes the total throughput. (A future direction to investigate is to iteratively optimize step i) and step ii) in an alternating manner, which remains an open problem.)

III. Optimal Uplink Channel Allocation

A. Facts from the Single Cell and the Two-Cell Cases

For each single cell, we have the following lemma:

Lemma 1 (Revenue Invariance): Given rdf, the revenue of *any* one user within a cell is invariant to different CAs that conform to this rdf.

This is an immediate implication of the flat fading and the in-cell orthogonalization assumptions.

For the multi-cell cases, the revenue-cost separation principle (Section II-B) and the revenue invariance property (Lemma 1) imply the following:

Corollary 1: To *maximize* the multi-cell total throughput given rdf of all cells, it is equivalent to find the optimal CA that has the *minimum total* $cost$ incurred.

For notational simplicity, whenever considering three consecutive cells on a line, we denote the center cell by C, the left cell by L, and the right cell by R (Figure 1). We denote the user indices occupying channel $j(= 1, \ldots, m)$ in L, C, and R by $L(j)$, $C(j)$, and $R(j)$. These *user index functions* can also be viewed as *channel allocation functions* because they fully specify the CAs in L, C and R.

Now, consider *two* adjacent cells L and C. We have the following lemma on the interference received by C from L.

Lemma 2: Given rdf of L, the *set of (a total of* m*) interference strengths* generated from the users in L to the BS of C is invariant to the actual CA of L.

This is again an immediate implication of the flat channel assumption. In fact, for any *one* user i in L, its (a total of $rdf(i)$) interference strengths seen at the BS of C do not depend on which $rdf(i)$ channels it occupies.

B. Interference Strength Alignment of the Two Neighboring Cells

For any three consecutive cells L, C and R, we now consider minimizing the *total* cost that C sustains, generated

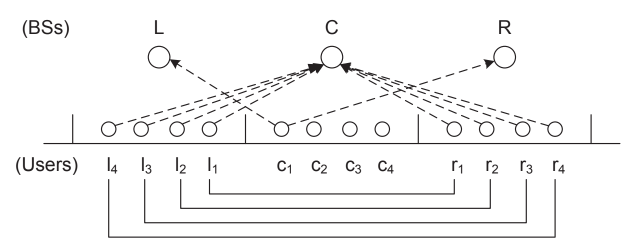

Fig. 1. Three consecutive cells in a linear network. Users connected by lines are suggested co-channel users.

from L and R. Clearly, the total cost that C sustains does not depend on its own CA, but depends on the CAs of L and R. According to Lemma 2, denote the set of interference strengths from L to C by $\{I_L^1, \ldots, I_L^m\}, I_L^1 \geq I_L^2 \ldots \geq I_L^m$. Due to the *flat* channel assumption, WLOG, we *index* the m channels such that I_L^j is the interference from L to C in channel j. Consequently, the user index function $L(j)(j = 1, \ldots, m)$ implied by such indexing satisfies that $I_L^j = g_{L(j)C(j)} P_{L(j)}$. (Note that receiver $C(j)$ is just the BS of C regardless of j.)

Next, we would like to specify the CA in R (i.e. $\{R(j), j = 1, \ldots, m\}$) under the given rdf, such that the total cost generated from $\{L \textit{ and } R\}$ to C is minimized.

We start with an arbitrary initial CA $R(j)$, and compute $\{I_R^j = g_{R(j)C(j)} P_{R(j)}, \ j = 1 \ldots m\}$. With the channel allocation functions $L(j)$ and $R(j)$, the total cost that the users in C sustain is

$$\sum_{j=1}^{m} cost^j = \sum_{j=1}^{m} \log(I_L^j + I_R^j + N), \tag{3}$$

where $cost^j$ is the interference cost that C sustains in channel j, and I_L^j and I_R^j $(j = 1, \ldots, m)$ are co-channel interferences.

For any other CA $R'(j)$ under the same rdf, it can be represented by a permutation function $P(j)(j = 1, \ldots, m)$ applied to the initial CA $R(j)$, such that $R'(j) = R(P(j))$. In other words, *channel j is assigned to the user who initially has channel $P(j)$ assigned to it.* With $L(j)$ and $R'(j)$, the total cost that the users in C sustain changes to

$$\sum_{j=1}^{m} cost'^j = \sum_{j=1}^{m} \log(I_L^j + I_R^{P(j)} + N), \tag{4}$$

where I_L^j and $I_R^{P(j)}$ $(j = 1, \ldots, m)$ are the new co-channel interferences.

The key in finding the optimal $P(j)$ (and hence the optimal CA in R) among the $m!$ permutations is an idea of *interference strength alignment*. We sort $\{I_R^j, j = 1, \ldots, m\}$ in descending order: $I_R^{j_1} \geq I_R^{j_2} \geq \ldots \geq I_R^{j_m}$, and *define* $P^*(k) \triangleq j_k, k = 1, \ldots, m$. The following theorem can be shown:

Theorem 1: Among all permutation functions $P(j)$, $\{P^*(j), j = 1, \ldots, m\}$ yields the *minimum* total cost that the users in C sustain.

Remark 1: From the definition of $P^*(j)$, we have $I_L^1 \geq I_L^2 \ldots \geq I_L^m$ and $I_R^{P^*(1)} \geq I_R^{P^*(2)} \geq \ldots \geq I_R^{P^*(m)}$. What has been done in the optimal CA is that we *align* the strongest interference each from L and R to co-exist in the same channel, and so on for the 2^{nd} strongest, …, all the way to aligning the weakest interference each from L and R to co-exist in the same channel.

Proof of Theorem 1: We prove an equivalent form of the theorem: If $a_1 \geq a_2 \geq \ldots \geq a_m \geq 0, b_1 \geq b_2 \geq \ldots \geq b_m \geq 0$, then $\sum_{j=1}^m \log(a_j + b_j + N) \leq \sum_{j=1}^m \log(a_j + b_{f(j)} + N)$, for all permutation functions $f(j), j = 1, \ldots, m$.

We use induction on m as follows.

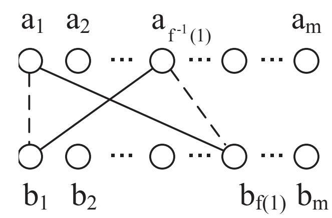

Fig. 2. After replacing $(a_1, b_{f(1)})$ and $(a_{f^{-1}(1)}, b_1)$ with (a_1, b_1) and $(a_{f^{-1}(1)}, b_{f(1)})$, the total cost decreases, (or remains unchanged.)

$m = 1$ is trivial. For $m = 2$, by Jensen's inequality,

$$\frac{a_1 - a_2}{a_1 - a_2 + b_1 - b_2} \log(a_1 + b_1 + N) + \frac{b_1 - b_2}{a_1 - a_2 + b_1 - b_2} \log(a_2 + b_2 + N) \leq \log(a_1 + b_2 + N) \tag{5}$$

$$\frac{b_1 - b_2}{a_1 - a_2 + b_1 - b_2} \log(a_1 + b_1 + N) + \frac{a_1 - a_2}{a_1 - a_2 + b_1 - b_2} \log(a_2 + b_2 + N) \leq \log(a_2 + b_1 + N) \tag{6}$$

(5)+(6) implies the theorem for $m = 2$.

Suppose the theorem holds for all $m \leq k$.

For $m = k + 1$, we first represent the problem by a bipartite graph (Figure 2): it consists of upper m points $a_1, \ldots, a_m$ and lower m points $b_1, \ldots, b_m$, and there is an (undirected) edge between every a_j and $b_{f(j)}$. The theorem is then equivalent to claiming that the graph with all "vertical" edges $(a_j, b_j), j = 1, \ldots, m$ yields the minimum total cost. Proof follows:

Given a graph generated from a permutation function $f(j)$:

i) If edge (a_1, b_1) is in the graph, then after removing (a_1, b_1), the rest of the graph degrades to an $m = k$ case, and applying the induction assumption proves this case.

ii) If edge (a_1, b_1) is not in the graph, Applying the induction assumption with $m = 2$ yields,

$$\log(a_1 + b_{f(1)} + N) + \log(a_{f^{-1}(1)} + b_1 + N) \geq \log(a_1 + b_1 + N) + \log(a_{f^{-1}(1)} + b_{f(1)} + N) \tag{7}$$

In other words, after replacing the two edges $(a_1, b_{f(1)})$ and $(a_{f^{-1}(1)}, b_1)$ with (a_1, b_1) and $(a_{f^{-1}(1)}, b_{f(1)})$, the total cost decreases (or remains unchanged), and the new bipartite graph falls into the case of i). ■

C. Optimal Channel Allocation in Infinite Linear Cellular Networks

Now, consider a two-sided infinite linear cellular network, with rdf of users in all cells given. The above alignment procedure can then be used in a rippling manner along the linear network to obtain the optimal CA of all cells — indexed as $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

We first minimize the total cost sustained by cells $\{\ldots, -3, -1, 1, 3, \ldots\}$: this leads to the optimal CA in cells $\{\ldots, -2, 0, 2, \ldots\}$, specified by the following procedure.

Step 0.	Assign an arbitrary CA to cell 0;
Step 1a.	Based on the CA in cell 0, assign the CA in cell 2 according to the the alignment rules in Theorem 1, and the total cost that cell 1 sustains is minimized;
Similarly,	
Step 1b.	CA in cell 0 $\Rightarrow$ CA in cell -2: the cost in cell -1 is minimized;
Step 2a.	CA in cell 2 $\Rightarrow$ CA in cell 4: the cost in cell 3 is minimized;
Step 2b.	CA in cell -2 $\Rightarrow$ CA in cell -4: the cost in cell -3 is minimized;
And so on.	

Note that in Step 0, the arbitrary CA assignment of cell 0 does not lose any generality (and hence optimality) due to the flat channel assumptions.

Next, we minimize the total cost sustained by cells $\{\ldots, -2, 0, 2, \ldots\}$ by applying the rippling procedure to cells $\{\ldots, -3, -1, 1, 3, \ldots\}$. Finally, we obtain a complete optimal CA that yields the *minimum total cost* of all the cells. Because of the revenue invariance property (Lemma 1), it achieves the *maximum total throughput*.

Remark 2: Finding a complete optimal CA has a complexity of $O(n_{cell} m \log(m))$, where n_{cell} is the number of cells: The term n_{cell} comes from the rippling procedure as above, and the term $m \log(m)$ comes from sorting the interference strengths for each cell (using e.g. Heapsort.)

Remark 3: For all the users in any one cell, the *ordering* of their interference strengths at the left neighboring BS could be *different* from that at the right neighboring BS. (E.g., in Figure 1, user c_1 (among all the users in C) creates the strongest interference to the BS of L, but the weakest interference to the BS of R.) To guarantee that a CA is *global* optimal, users within each cell must align their interference strengths with those from *both* their 2^{nd} left and 2^{nd} right neighboring cells, (as achieved by the proposed rippling procedure.)

Remark 4: We have considered linear cellular networks. Generalizations to finding the optimal CA in two dimensional cellular networks also follow the interference strength alignment idea. Each cell, however, needs to align with more than two other cells (instead of aligning with just the 2^{nd} left and the 2^{nd} right neighboring cells as in the linear case.) This often leads to the *impossibility of perfect alignment* of all the cells, and significantly raises the complexity of finding the global optimal CA.

IV. Performance Evaluation: Optimal CA vs. CDMA

In this section, we give two typical examples that numerically compare the *average spectral efficiency* of the optimal CA with that of CDMA schemes. We assume the simplified path loss model [9] $P_r = P_t K (\frac{d_0}{d})^\gamma$ with $d_0 = 50m$ (outdoor environment), and no multipath or shadow fading. The parameter K is irrelevant to the comparison between optimal CA and CDMA, and is assumed to be 1. γ between 2.5 and 4 will be tested below. We assume a linear cellular network with cell radius $= 500m$, and that there are n users equally spaced in every cell.

Example 1. Users with Equal Resource Share — Uniform rdf

In this example, we assume that every user gets the same unit channel resource (uniform rdf,) and all users transmit at the same uniform PSD. We compute a completely interference limited case, i.e., noise power is ignored. In this case, the value of the PSD is irrelevant to the comparison between optimal CA and CDMA, and is assumed to be 1. For any three consecutive cells L, C, R, the optimal CA derived in Section III yields a fully symmetric CA in L and R such that users in L and R with the same distance to the BS of C co-exist in the same channel (Figure 1). With the uniform rdf, the average cost per user is

$$\overline{cost}_{\mathsf{opt.CA}} = \frac{1}{n}\sum_{i=1}^{n} \log(2I_i) \quad \text{bits/sec/Hz} \tag{8}$$

where I_i $(i = 1, \ldots, n)$ traverses the interference from all n positions of the users in L (and R symmetrically.)

With CDMA, every user in a cell sustains the *same* amount of cost from other cells (since interference is averaged:)

$$\overline{cost}_{\mathsf{CDMA}} = \log(\frac{1}{n}\sum_{i=1}^{n} 2I_i) \quad \text{bits/sec/Hz} \tag{9}$$

From Jensen's inequality, the superiority of the optimal CA over CDMA becomes evident: $\overline{cost}_{\mathsf{CDMA}} \geq \overline{cost}_{\mathsf{opt.CA}}$ always. We plot in Figure 3 the difference $\overline{cost}_{\mathsf{CDMA}} - \overline{cost}_{\mathsf{opt.CA}}$ as a function of n, parameterized by $\gamma = 2.5, 3, 3.5, 4$. Note that from revenue invariance, $-(\overline{cost}_{\mathsf{CDMA}} - \overline{cost}_{\mathsf{opt.CA}})$ equals the throughput difference in terms of the average spectral efficiency. We observe the following:

1) As the number of users increases and/or as γ increases, the *variation* in the set of interference strengths $\{I_i\}$ increases. Thus the gap from Jensen's inequality, and hence the cost difference, increases.

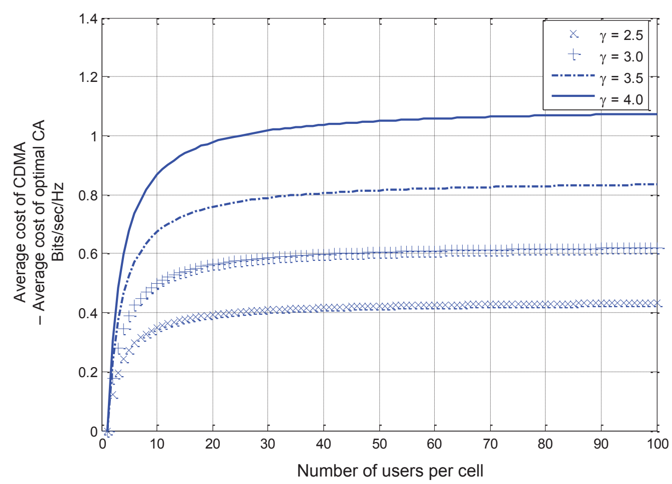

Fig. 3. Superiority of optimal CA over CDMA — Equal resource case

2) As one numerical rule of thumb in this particular example, with $\gamma = 4$, the superiority in throughput of optimal CA over CDMA reaches above 1 bits/sec/Hz when the number of users reaches 25 per cell.

Example 2. Users with Equal Rate — Hard Fairness

In this example, we guarantee that users at all positions in a cell achieve the same rate in bits/sec. Thus, rdf needs to be designed such that the edge users have more bandwidth. We achieve this by designing rdf such that every user in a cell has the same *total revenue* in *bits/sec*. This requires $rdf(i)$ being *inversely proportional to* $revenue_i$ in *bits/sec/Hz* (we assume that the uniform transmit PSD of all users is set such that a user at the cell edge has a $revenue$ of 1 bit/sec/Hz.) Next, equal *total cost* that each user sustains can be achieved by properly time sharing different optimal CAs. At the end, each user achieves an equal rate in bits/sec.

We first normalize rdf within a cell: $rdf(i) \leftarrow \frac{rdf(i)}{\sum_{j=1}^{n} rdf(j)}$, such that $\sum_{i=1}^{n} rdf(i) = 1$. Then, similarly to Example 1, the average cost with optimal CA is

$$\overline{cost}'_{\mathsf{opt.CA}} = \sum_{i=1}^{n} \log(2I_i + N) rdf(i) \quad \text{bits/sec/Hz} \tag{10}$$

With CDMA, the cost every user sustains is again equal to each other:

$$\overline{cost}'_{\mathsf{CDMA}} = \log(\sum_{i=1}^{n} 2I_i rdf(i) + N) \quad \text{bits/sec/Hz} \tag{11}$$

Clearly, (10) (11) are generalizations of (8) (9): when rdf is a uniform function, (with N ignored,) (10) (11) degrade into (8) (9).

Again, from Jensen's inequality, $\overline{cost}'_{\mathsf{CDMA}} \geq \overline{cost}'_{\mathsf{opt.CA}}$ always. In addition to the previous observations in Example 1 which appear similarly here, we have computed and observed that requiring hard fairness makes *the throughput difference of optimal CA over CDMA even larger* — The asymptotic differences increase almost 50% compared to the equal resource share case. The intuition is that allocating more bandwidth to edge users increases the "total variation" in the set of interference strengths, and hence the gap from Jensen's inequality.

Remark 5: We have used rdf to achieve fairness between users. We note that while using power control (PC) is also an option, preliminary investigation has shown that *with CDMA*, PC is often less efficient than adjusting rdf. The intuition is that to compensate cell-edge users' path loss disadvantage, raising their power (instead of allocating more channels to them) often creates more aggregate interference to neighboring cells. For the optimal CA problem, the comparison between adjusting rdf and PC remains an open question.

V. Conclusion

We defined the resource distribution function (rdf) which takes care of the fairness requirement in cellular networks. Under any given rdf as a fairness constraint, we have found the uplink channel allocation that maximizes the total throughput in linear cellular networks, assuming flat fading and that users have fixed power levels (but not necessarily equal to each other.) The optimal channel allocation is characterized by an interference strength alignment property — for every cell, its neighboring cells' interference in all m channels must be aligned such that each neighboring cell's i^{th} strongest interference ($i = 1, \ldots, m$) co-exists in the same channel. The complete optimal channel allocation is found with $O(n_{cell} m \log(m))$ complexity. From Jensen's inequality, our results provide a clear justification of the superiority of optimal channel allocation over CDMA in terms of throughput. Numerical comparisons under various fairness requirements in linear cellular networks have shown evident gaps in bits/sec/Hz between the throughput of optimal CA and that of CDMA.

For two (or higher) dimensional cellular networks in frequency selective channels, finding the global optimal uplink CA remains open due to its high complexity. Future research will also be pursued on the combination of power allocation and rdf design with the proposed channel allocation algorithm.

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