Optimal Spectrum Management in Multiuser Interference Channels

Yue Zhao, and Gregory J. Pottie
Department of Electrical Engineering
University of California, Los Angeles
Los Angeles, CA, 90095, USA
Email: yuezhao@ucla.edu, pottie@ee.ucla.edu

***Abstract*—In this paper, we investigate the optimal spectrum management problem in multiuser frequency selective interference channels. First, a simple pairwise interference coupling condition under which FDMA can achieve all Pareto optimal points of the rate region is discovered. Not only is this condition sufficient, we show that it is also necessary for FDMA to be always optimal at least in symmetric channels. For the general cases where this condition is not necessarily satisfied, we first explicitly obtain the optimal solution as the optimal combination of flat FDMA and flat frequency sharing for the sum-rate maximization problem in two user symmetric flat channels, and then show that the general *n*–user weighted sum-rate maximization in non-symmetric frequency selective channels can be formulated into primal domain convex optimizations.**

I. INTRODUCTION

We consider the scenario of multiple multicarrier communications systems contending in a common frequency band, in which interference coupling between different users remains a major problem that limits the multiuser performance. We investigate the optimal spectrum and power allocation that achieves any Pareto optimal point of the achievable rate region, under the assumption that interference is treated as noise at the receivers.

There are essentially two strategies for multiple users to co-exist: *FDMA and frequency sharing (overlapping)*. As the cross coupling varies from being extremely strong to extremely weak, the preferable co-existence strategies intuitively shift from complete avoidance (FDMA) to pure frequency sharing. We start from the strong coupling scenario, and investigate the weakest interference condition under which FDMA is still guaranteed to be optimal. In the literature, a relatively strong *pairwise* coupling condition for FDMA to be optimal was proved, and it applies to all Pareto optimal points of the *n*-user rate region [5]. By pairwise we mean that whether two users should avoid each other only depends on the interference condition between those two users. For one typical Pareto optimal point which is the *sum-rate* maximization point, the required coupling strengths for FDMA to be optimal are further lowered [6]. However, this condition is a *group-wise* one, meaning that the couplings between *all* existing users are required to be strong for FDMA to be provably sum-rate optimal.

We relax these conditions and obtain the weakest possible pairwise condition for FDMA to be optimal: for any two users, as long as the two normalized cross couplings between them are both larger than or equal to 1/2, *all* n-user Pareto optimal points are guaranteed to be achievable with FDMA between these two users. When the interference coupling is less than 1/2 in symmetric channels, we give a precise characterization of the non-empty power constraint region within which frequency sharing between two users leads to a higher rate than an FDMA between them. Thus, the proposed condition for FDMA to be always optimal is not only sufficient, but also necessary.

With the interference coupling less than 1/2, the weighted sum-rate maximization is in the form of a non-convex optimization and generally hard to solve [9]. However, the Lagrangian dual problem is decomposed in frequency and easier to solve [4][10]. It is shown in the literature that the duality gap goes to zero when the number of sub-channels goes to infinity [10]. This justifies the asymptotic optimality of solving the problem in the dual domain, and many spectrum balancing algorithms using dual methods have been developed [3] [4] [10].

We approach this general non-convex optimization from the primal perspective. We start with the sum-rate maximization problem in two-user symmetric flat channels, and obtain analytically the optimal solution by combining FDMA and frequency sharing in an optimal way. By generalizing this method, we show that all the general *n*-user arbitrarily weighted sum-rate maximization in non-symmetric frequency selective channels can be formulated into equivalent *primal domain convex optimizations*. As will be shown at the end, it also directly implies the zero duality gap theorem in the literature [10]. In retrospect, the methodology we provide shares some common insight with the time sharing condition discussed in [10].

TABLE I
PROBLEMS, PRIOR WORK, AND RELATED SECTIONS IN THIS PAPER

Problems				Prior Work	Our Results
Spectrum Management in Cooperative Scenarios					
	Strong Interference Scenarios	Conditions for FDMA schemes to be optimal		[5][6]	Section III
		Finding optimal schemes with FDMA constraints		[6][8]	
	General Interference Scenarios	Continuous Frequency Scenarios	Primal domain solution: Equivalent Convex Formulation		Section IV
			Dual domain methods	[10]	
		Discrete Frequency Scenarios: Approximation Algorithms		[3][4][7][10]	
Spectrum Sharing in Non-cooperative Scenarios: Nash Equilibriums				[5][9]	

Table I summarizes the various forms of the multiuser interference channel co-existence problems, the prior work, and in which sections we present solutions that improve upon these prior results. We suggest future research directions in the conclusion. Due to space limitations, proof details which can be found in [11] are omitted here in favor of explaining the sequence of results and their significance.

II. Channel Model and Two Basic Co-existence Strategies

An n-user interference channel is modeled by $y_i = H_{ii}x_i + \sum_{j\neq i} x_j H_{ji} + n_i,\ i = 1,2,...,n$, where x_i is the transmitted signal of user i, and y_i is the received signal of user i including additive Gaussian noise n_i (a user corresponds to a pair of transmitter and receiver). H_{ii} are the direct channel gains, whereas H_{ji} are the cross coupling gains. We assume that the channel is frequency selective over the band $(f_1,\ f_2)$, where $W \triangleq f_2 - f_1$ is the total bandwidth. The channel gains are denoted by $H_{ii}(f)$ and $H_{ji}(f)$. The transmit power spectrum density (PSD) of user i is denoted by $P_i(f)$, and the noise PSD at receiver i by $\sigma_i(f)$. We assume that interference is treated as noise and random Gaussian codebooks are used. The achievable rate for user i is $R_i = \int_{f_1}^{f_2} \log\left(1 + \frac{P_i(f)\left|H_{ii}(f)\right|^2}{\sigma_i(f) + \sum_{j\neq i} P_j(f)\left|H_{ji}(f)\right|^2}\right) df$.

Normalizing the channel gains and noise power by the direct channel gains, we have

$$R_i = \int_{f_1}^{f_2} \log\left(1 + \frac{P_i(f)}{N_i(f) + \sum_{j\neq i} P_j(f)\alpha_{ji}(f)}\right) df,$$

where $N_i(f) \triangleq \frac{\sigma_i(f)}{\left|H_{ii}(f)\right|^2}$ and $\alpha_{ji}(f) \triangleq \frac{\left|H_{ji}(f)\right|^2}{\left|H_{ii}(f)\right|^2}$.

To facilitate analyzing the optimal spectrum management scheme, we introduce two basic co-existence strategies: *Flat Frequency Sharing* and *Flat FDMA*, both defined in flat channels. These two strategies are the building blocks of all non-flat co-existence strategies in frequency selective channels, and will be used to establish general conditions for FDMA to be optimal.

Consider a two-user flat channel: $\forall f \in (f_1,\ f_2)$,

$$N_1(f) = n_1, N_2(f) = n_2, \alpha_{21}(f) = \alpha_{21}, \alpha_{12}(f) = \alpha_{12}, \qquad (1)$$

a flat frequency sharing scheme of two users is defined as any power allocation in the form of

$$P_1(f) = p_1,\ P_2(f) = p_2,\ \forall f \in (f_1,\ f_2); \qquad (2)$$

a flat FDMA scheme of two users is defined as any power allocation in the form of

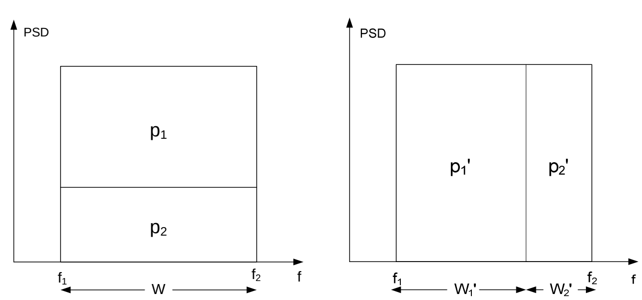
$$P_1(f)P_2(f) = 0 \ \text{ and } \ P_1(f) + P_2(f) = p,\ \forall f \in (f_1, f_2).$$

Next, we define the *flat FDMA reallocation* to be the following *power invariant* transform that reallocates a flat frequency sharing scheme to be a flat FDMA scheme: user 1 reallocates its power within a sub-band $W_1' = (p_1 / p_1 + p_2)W$ with a flat PSD $p_1' = p_1 + p_2$; user 2 reallocates its power within another disjoint sub-band $W_2' = (p_2 / p_1 + p_2)W$ with the same flat PSD $p_2' = p_1 + p_2$.

Illustrations of the power allocations of the two basic co-existence strategies before and after a flat FDMA reallocation are depicted in Fig. 1. Similarly, flat frequency sharing schemes, flat FDMA schemes, and flat FDMA reallocation in n-user flat channel cases can be defined.

III. Strong interference scenario: The Conditions for the Optimality of FDMA

In this section, we investigate the conditions under which



Flat Frequency Sharing
(Before flat FDMA reallocation)

Flat FDMA
(After flat FDMA reallocation)

Fig. 1. Power allocations of flat frequency sharing and flat FDMA, also an illustration of flat FDMA reallocation.

the optimal spectrum and power allocation is FDMA, and our objective is to encompass all Pareto optimal points. Firstly, we show a coupling condition under which FDMA is optimal within a group of strongly coupled users. We then show that this coupling condition also works when there are other users that are not strongly enough coupled. The two basic co-existence strategies serve as a powerful tool in proving the general condition for the optimality of FDMA.

We begin with two-user flat channels.

Theorem 1: Consider a two-user flat interference channel (1). Suppose the two users co-exist in a flat frequency sharing manner (2). If $\alpha_{12} \ge 1/2$ *and* $\alpha_{21} \ge 1/2$, then with a flat FDMA power reallocation, *both* users' rates will be higher (or unchanged).

Proof : See [11], section III.A. ■

Theorem 1 can be generalized to *n*-user cases in frequency selective channels [11]. We summarize these results as follows: pick any sub-band (f_1', f_2'), as long as all the users having power within this sub-band are strongly coupled with $\alpha_{ji}(f) \ge 1/2, \quad \forall j \ne i, \ \forall f \in (f_1', f_2')$, then for any power allocation scheme having frequency sharing happening anywhere within this sub-band, there always exists an FDMA power reallocation scheme (with the total power unchanged for each user) that leads to a rate higher than or equal to the original sharing scheme for *every existing user*.

We have shown the condition for FDMA schemes to be optimal within strongly coupled users. In real communication networks, however, there are usually users not strongly enough (maybe just moderately) coupled with some other users. For these users outside the strongly coupled group, we show that *they always benefit from an FDMA within the strongly coupled group*.

We begin with two-interferer flat channels.

Theorem 2: Consider a three-user (one user + two interferers) flat channel: $N_i(f) = n_i$, $\alpha_{ji}(f) = \alpha_{ji}$. Suppose the three users co-exist in a flat frequency sharing manner: $P_i(f) = p_i, \quad \forall f \in (f_1, f_2), \ i = 0,1,2.$ From user 0's perspective, a flat FDMA power reallocation of its two interferers user 1 and user 2 always leads to a higher (or equal) rate for user 0.

Proof: See [11], section III.B. ■

Theorem 2 can be generalized to an arbitrary number of users in frequency selective channels, proving that an FDMA within a subset of users is always preferred by every user who is not in this subset, and this is true for all coupling conditions [11]. In the case that $\exists i, j, \ \alpha_{ji}(f) \ge 1/2$ and $\alpha_{ij}(f) \ge 1/2$, combining Theorem 1 and Theorem 2 gives us a very strong insight into the conditions under which the optimal co-existence strategies must be FDMA: Suppose there are $n(\ge 2)$ users, for any two users *i* and *j* among them, for any frequency band (f_1', f_2'), if the normalized cross coupling gains $\alpha_{ji}(f) \ge 1/2$ and $\alpha_{ij}(f) \ge 1/2$, $\forall f \in (f_1', f_2')$, then *no matter from which of the n users' point of view*, an FDMA of user *i* and user *j* within this band is always preferred.

This pairwise condition is very convenient to use because it makes determining whether any two users should be orthogonally channelized depend only on the coupling conditions between the two of them. On the other hand, since this condition guarantees that an FDMA between user *i* and user *j* benefits every existing user, we conclude (with an immediate proof by contradiction) that under this condition, all the Pareto optimal points of the *n*-user achievable rate region can be achieved with these two users being orthogonalized (FDMA). In this section, we have shown that this condition is sufficient. In Section IV, we show that it is also necessary, i.e. it cannot be further weakened.

IV. General Interference Scenario: Optimal Spectrum Management in Frequency Selective Channels

In this section, we continue to analyze the optimal spectrum management in the cases in which $\alpha(f)$ can be less than 1/2. We first analyze two-user symmetric flat channels, and then extend our results to the general cases. (Similar results for the symmetric flat channel case have also been independently developed in [1].)

A. Solution of Sum-rate Maximization in Two-user Symmetric Flat Channels with Equal Power Constraints

Consider the sum-rate maximization problem in a two-user symmetric flat Gaussian interference channel:

$$\alpha_{12}(f) = \alpha_{21}(f) = \alpha, \ N_1(f) = N_2(f) = n, \forall f \in (f_1, f_2) \quad (3)$$

First, we have the following theorem on the sufficient and necessary condition for a flat FDMA scheme to be better than a flat frequency sharing scheme.

Theorem 3: For any flat frequency sharing power allocation, a flat FDMA power reallocation (Fig. 1) leads to a higher or unchanged sum-rate if and only if $(p_1 + p_2)/n \ge 2\left(1/2\alpha^2 - 1/\alpha\right)$.

Proof: See [11], section VI.A. ■

Define the critical point $p_0 = 2\left(1/2\alpha^2 - 1/\alpha\right)$. Clearly, when $\alpha < 1/2, \ \ p_0 > 0$, and within the non-empty triangular power region $0 < p_1 + p_2 < np_0$ flat frequency sharing is better than flat FDMA (which is the optimal FDMA scheme in flat channels). It thus shows the necessity of the condition $\alpha \ge 1/2$ for FDMA to be always optimal.

Next, we impose an equal power constraint *P/2* for both users. The optimization problem becomes:

$$\max \ R_1 + R_2, \ s.t. \ \int_{f_1}^{f_2} P_i(f) df \le \frac{P}{2}, \ P_i(f) \ge 0, \ i = 1,2. \quad (4)$$

We normalize the signal power by the noise power and let $n = 1$. Define an average density $p = P/W$. Then the maximum achievable sum-rate with flat frequency sharing is

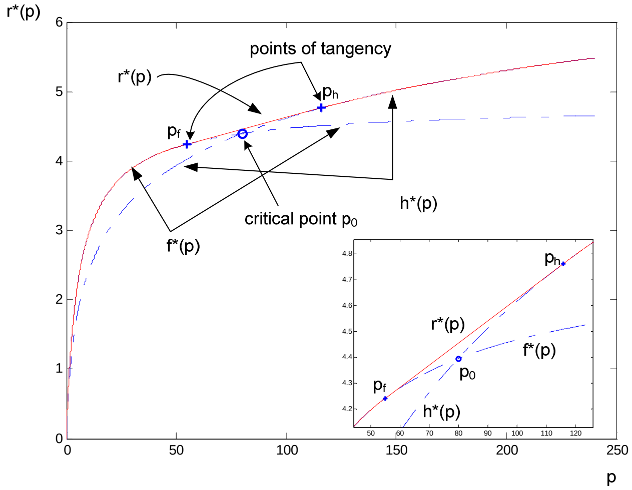


Fig. 2. The maximum achievable sum-rate as the convex hull of the rates of flat FDMA and flat frequency sharing, $\alpha = 0.1$.

$f^*(p) = 2W\log\left(1+\dfrac{p/2}{1+\alpha p/2}\right)$ [11]. The maximum achievable sum-rate with FDMA is $h^*(p) = W\log(1+p)$.

Define $r(p) = \max(f^*(p), h^*(p))$. It can be verified that $f^*(p)$ and $h^*(p)$ intersects at $p = p_0$ (directly implied by Theorem 3), and $r(p)$ is *not* concave in $[0,\infty)$. Next, define $r^*(p)$ to be the unique *convex hull* of $r(p)$. A typical plot of $f^*(p)$, $h^*(p)$, and the convex hull $r^*(p)$ when $\alpha < 1/2$ is given in Fig. 2. When the power constraint *p* falls between the two points of tangency p_f and p_h on the convex hull, $r^*(p)$ can be achieved by applying flat FDMA and flat frequency sharing in disjoint sub-bands respectively [11]. Next, we show in the following theorem that $r^*(p)$ is not only achievable, but also an *upper bound* (Proof by Jensen's inequality in continuous frequency [11]), and hence optimal.

Theorem 4: In a two-user symmetric flat Gaussian interference channel (3), the maximum achievable sum-rate with power constraint *P/2* (4) for both users is $r^*(p)$.

Proof: See [11], section VI.A. ■

The major implications of Theorem 4 are as follows [11]:

i) The maximum achievable sum-rate in this case is computable. In fact, after solving the two points of tangency as in Fig. 2, we have an analytic expression for the maximum sum-rate as a concave function of the power constraint.

ii) The optimal (potentially frequency selective) spectrum and power allocation is a combination of *flat* frequency sharing and *flat* FDMA in two disjoint bands, combined according to where the power constraint *p* lies on the curve of $r^*(p)$.

iii) In flat channels, the convex hull of any achievable sum-rate function (as a function of power constraints) is also achievable.

B. Primal Domain Convex Optimization Formulation for General Frequency Selective Interference Channels

We now consider the general weighted sum-rate maximization in *n*-user non-symmetric frequency selective channels with arbitrary individual power constraints:

$$\begin{aligned}&\max_{P_i(f), i=1,2,\ldots,n} \sum_{i=1}^{n} w_i R_i,\\ &s.t. \int_{f_1}^{f_2} \mathbf{P}(f)df \le \mathbf{P},\ \ \mathbf{P}(f)\ge 0,\ \forall f\in(f_1,f_2)\end{aligned} \tag{5}$$

where $\mathbf{P}(f) = (P_1(f),\ldots,P_n(f))$, and the power constraints are $\mathbf{P} = (P_1,\ldots,P_n)$. Define the *rate density function* as

$$r(\mathbf{P}(f); f) \triangleq \sum_{i=1}^{n} w_i \log\left(1+\frac{P_i(f)}{N_i(f)+\sum_{j\neq i} P_j(f)\alpha_{ji}(f)}\right).$$

Problem (5) can then be rewritten as

$$\begin{aligned}&\max_{P_i(f), i=1,2,\ldots,n} \int_{f_1}^{f_2} r(\mathbf{P}(f); f)df\\ &s.t. \int_{f_1}^{f_2} \mathbf{P}(f)df \le \mathbf{P},\ \ \mathbf{P}(f)\ge 0,\ \forall f\in(f_1,f_2)\end{aligned} \tag{6}$$

At every frequency point *f*, $r(\mathbf{P}(f); f)$ is a non-concave function of $\mathbf{P}(f)$, making optimization non-convex and hard. Now, we define $r^*(\mathbf{P}(f); f)$ as the convex hull of $r(\mathbf{P}(f); f)$ along the *n* dimensions of users' power:

1) Define the set of functions
$S = \{\tilde{r}(\mathbf{P}(f); f) \mid \tilde{r}(\mathbf{P}(f); f)$ concave in $\mathbf{P}(f)$;
$\tilde{r}(\mathbf{P}(f); f) \ge r(\mathbf{P}(f); f),\ \forall \mathbf{P}(f)\ge 0\}$

2) $r^*(\mathbf{P}(f); f)$ is the unique function satisfying

$$\begin{cases} r^*(\mathbf{P}(f); f) \in S \\ r^*(\mathbf{P}(f); f) \le \tilde{r}(\mathbf{P}(f); f),\ \forall \mathbf{P}(f)\ge 0,\ \forall \tilde{r}(\mathbf{P}(f); f)\in S \end{cases}$$

Replacing the original non-concave rate density function $r(\mathbf{P}(f); f)$ in (6) by its convex hull $r^*(\mathbf{P}(f); f)$ at every frequency point *f*, we obtain the following *convex* optimization:

$$\begin{aligned}&\max_{P_i(f), i=1,2,\ldots,n} \int_{f_1}^{f_2} r^*(\mathbf{P}(f); f)df\\ &s.t. \int_{f_1}^{f_2} \mathbf{P}(f)df \le \mathbf{P},\ \ \mathbf{P}(f)\ge 0,\ \forall f\in(f_1,f_2)\end{aligned} \tag{7}$$

Clearly, (7) has an optimal value that upper bounds that of the original problem (6), because the convex hull $r^*(\mathbf{P}(f); f)$ upper bounds $r(\mathbf{P}(f); f)$ itself at every frequency point *f*. On the other hand, by treating every frequency point as an infinitesimal flat channel, any achievable objective value for (7) is also achievable for the original non-convex one (6). We then have the following theorem:

Theorem 5: The convex optimization (7) has the same optimal value as the original non-convex optimization (6).

Proof: See [11], section V. ■

From Theorem 5, we see that the original non-convex optimization (6) can be transformed in the primal domain to convex optimization (7) without loss of optimality. The optimal spectrum and power allocation of (7) can be transformed to that of (6) according to a weighting function with which the points on $r(\mathbf{P}(f); f)$ are weighted averaged (convexly combined) to be those on $r^*(\mathbf{P}(f); f)$ [11]. Furthermore, we now show that Theorem 5 directly leads to the zero duality gap result in the literature [10].

For the primal problem (6), the Lagrange dual is $L(\mathbf{P}(f), \boldsymbol{\lambda}) = \int_{f_1}^{f_2} r(\mathbf{P}(f); f)df - \boldsymbol{\lambda}^T(\int_{f_1}^{f_2} \mathbf{P}(f)df - \mathbf{P})$.

The dual objective is $g(\boldsymbol{\lambda}) = \sup_{\mathbf{P}(f)\geq 0} L(\mathbf{P}(f), \boldsymbol{\lambda})$.

For the primal problem (7), the Lagrange dual is $\hat{L}(\mathbf{P}(f), \boldsymbol{\lambda}) = \int_{f_1}^{f_2} r^*(\mathbf{P}(f); f)df - \boldsymbol{\lambda}^T(\int_{f_1}^{f_2} \mathbf{P}(f)df - \mathbf{P})$.

The dual objective is $\hat{g}(\boldsymbol{\lambda}) = \sup_{\mathbf{P}(f)\geq 0} \hat{L}(\mathbf{P}(f), \boldsymbol{\lambda})$.

Since $r^*(\mathbf{P}(f); f) \geq r(\mathbf{P}(f); f),\ \forall \mathbf{P}(f), \forall f$, we have $\hat{L}(\mathbf{P}(f), \boldsymbol{\lambda}) \geq L(\mathbf{P}(f), \boldsymbol{\lambda})$ and $\hat{g}(\boldsymbol{\lambda}) \geq g(\boldsymbol{\lambda})$ always. Denote the primal optimal values for (6) and (7) by p^* and $\hat{p}^*$, and the dual optimal values by $d^* = \min\ g(\boldsymbol{\lambda})\ s.t.\ \boldsymbol{\lambda} \geq 0$ and $\hat{d}^* = \min\ \hat{g}(\boldsymbol{\lambda})\ s.t.\ \boldsymbol{\lambda} \geq 0$. From $\hat{g}(\boldsymbol{\lambda}) \geq g(\boldsymbol{\lambda}),\ \ \forall \boldsymbol{\lambda} \geq 0$, we get $\hat{d}^* \geq d^*$. From weak duality for (6), $p^* \leq d^*$. Since problem (7) is a convex optimization, it has strong duality $\hat{p}^* = \hat{d}^*$ [2]. Thus $\hat{p}^* = \hat{d}^* \geq d^* \geq p^*$. From Theorem 5, (6) and (7) are equivalent, and thus $\hat{p}^* = p^*$. Therefore, the original non-convex optimization (6) also have strong duality (zero duality gap), i.e. $p^* = d^*$.

V. Concluding Remarks

In this paper, we have analyzed the optimal spectrum and power allocation in all coupling conditions. We have shown that for any two users, as long as the two normalized cross couplings between them are both larger than or equal to 1/2, an FDMA between these two users benefits every existing user, and hence can be used to achieve any Pareto optimal point of the *n*-user achievable rate region. Because this interference condition has a pairwise nature, it leads foreseeably to distributed implementation.

This condition cannot be further lowered as shown in two user symmetric flat channels. For the sum-rate maximization problem in this case with equal power constraints, we analytically obtained the optimal spectrum and power allocation which has a clear intuition of combining flat FDMA and flat frequency sharing in an optimal way. For the general *n*-user weighted sum-rate maximization problems in frequency selective channels, we generalized our insight from the flat channel case and formulated the originally non-convex optimization into an equivalent primal domain convex optimization by replacing the non-concave objective function at every frequency point with its convex hull. This result provides the performance limit and a new perspective into optimal algorithm designs in spectrum management.

This paper has worked on the continuous frequency domain problems, and hence has infinite-dimension variables. The ideas can be applied to discrete frequency spectrum management via approximation. With the new insights we obtained for this optimization problem, the design of novel practical algorithms to approach the optimal spectrum management is an interesting future research direction. We note in particular that while it is in general difficult to compute high dimensional convex hull functions, preliminary investigations reveal that either pure FDMA or pure sharing strategies suffer little loss compared to the optimal scheme over a considerable range of the coupling parameter α. Consequently, relatively simple strategies and algorithms may suffice in practice.

References

[1] Bhaskaran, S.R.; Hanly, S.V.; Badruddin, N.; Evans, J.S., "Maximizing the sum rate in symmetric networks of interfering links", Information Theory and Applications Workshop, 2009, Feb. 8 2009 - Feb. 13 2009

[2] Boyd, S.P., Vandenberghe, L., "Convex Optimization," Cambridge University Press, 2004.

[3] Cendrillon, R.; Jianwei Huang; Mung Chiang; Moonen, M., "Autonomous Spectrum Balancing for Digital Subscriber Lines," Signal Processing, IEEE Transactions on , vol.55, no.8, pp.4241-4257, Aug. 2007.

[4] Cendrillon, R.; Wei Yu; Moonen, M.; Verlinden, J.; Bostoen, T., "Optimal multiuser spectrum balancing for digital subscriber lines," Communications, IEEE Transactions on , vol.54, no.5, pp. 922-933, May 2006.

[5] Etkin, R.; Parekh, A.; Tse, D., "Spectrum sharing for unlicensed bands," Selected Areas in Communications, IEEE Journal on , vol.25, no.3, pp.517-528, April 2007.

[6] Hayashi, S.; Zhi-Quan Luo, "Spectrum Management for Interference -Limited Multiuser Communication Systems," Information Theory, IEEE Transactions on , vol.55, no.3, pp.1153-1175, March 2009.

[7] Chee Wei Tan; Palomar, D.P.; Mung Chiang, "Solving nonconvex power control problems in wireless networks: low SIR regime and distributed algorithms," Global Telecommunications Conference, 2005. GLOBECOM '05. IEEE , vol.6, no., pp. 6 pp.-, 28 Nov.-2 Dec. 2005.

[8] Wei Yu; Cioffi, J.M., "FDMA capacity of Gaussian multiple-access channels with ISI," Communications, IEEE Transactions on , vol.50, no.1, pp.102-111, Jan 2002.

[9] Wei Yu; Ginis, G.; Cioffi, J.M., "Distributed multiuser power control for digital subscriber lines ," Selected Areas in Communications, IEEE Journal on , vol.20, no.5, pp.1105-1115, Jun 2002.

[10] Wei Yu; Lui, R., "Dual methods for nonconvex spectrum optimization of multicarrier systems," Communications, IEEE Transactions on, vol.54, no.7, pp. 1310-1322, July 2006.

[11] Yue Zhao; Pottie, G.J., "Optimal Spectrum Management in Multiuser Interference Channels" (November 21, 2008). Center for Embedded Network Sensing. Technical Reports. Paper 64. http://repositories.cdlib.org/cens/techrep/64.