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Wind Aggregation Via Risky Power Markets

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***Abstract*—Aggregation of diverse wind power sources can effectively reduce their uncertainty, and hence the cost of wind energy integration. A risky power contract is proposed, by which wind power producers (WPPs) can trade uncertain future power for efficient wind aggregation. A two-settlement market with both the risky power contract and a conventional firm power contract is shown to have a unique competitive equilibrium (CE), characterized in closed form. The marginal contribution and diversity contribution of each WPP to the group of all WPPs are fairly reflected in the profit earned by this WPP at the CE. Moreover, the CE achieves the same total profit as achieved by a grand coalition of WPPs. In a coalitional game setting, the profit allocation induced by the CE is always in the core, and is achieved via a non-cooperative risky power market. The benefits of the risky power market are demonstrated using wind generation and locational marginal price data for ten WPPs in the PJM interconnection.**

***Index Terms*—Coalitional game, competitive equilibrium, power market, renewable energy integration, wind aggregation.**

I. Introduction

A MAJOR re-thinking in the design and operation of power markets is necessary to enable massive integration of wind energy resources into the electric grid [2]–[4]. California, for example, anticipates 33% renewable penetration by 2020, within which wind energy will play a crucial role. Conventionally, the grid operation procedures are designed for *small uncertainty scenarios* [5], [6]. Under these scenarios, operating reserves, typically supplied by *expensive fast-ramping fuel-based generators*, are scheduled to compensate for forecast errors in the load, which are often as low as $1\% - 3\%$. Wind power generation is, however, non-dispatchable and difficult to forecast several hours or more in advance [7]. To accommodate the uncertainty brought into the system by the increasing wind penetration, additional reserve capacity is needed (see, e.g., [8]), and adds significantly to the system cost. Effectively reducing the uncertainty of wind power generation is thus key to efficient wind energy integration.

Different market structures have been proposed for wind integration. One commonly seen approach (used in, e.g., California and Germany) is to take all wind power generation into the system as negative load via extra-market procedures such as feed-in tariffs [9], [10]. This approach is, however, not likely to be sustainable in the long run when the wind penetration level is high. This is because the cost of increased reserve margin socialized among load serving entities (LSEs) can become excessively large, and hence discourages LSEs to accept high wind penetration. A primary alternative approach, carried out in the U.K. for example, in essence requires wind power producers (WPPs) to participate in conventional electricity wholesale markets, and imposes a financial penalty on WPPs for their generation deviations from contracts offered in the forward market [11]. Such a market structure provides a strong incentive for WPPs to *firm* their own wind power generation, that is to reduce the generation uncertainty and variability, via a range of technical and market options. Well-studied examples include installing energy storage which mitigates uncertainty by shifting energy over different time periods, and improving the quality of forecast which effectively reduces the uncertainty level of a given wind power plant (cf. [12]–[14] for renewable integration with storage, and [15] for the role of forecast accuracy in dispatch with wind power sources). These approaches, however, have their own limits and are suitable only for certain systems. In particular, the current capital costs of storage make it difficult to deploy massive amounts of storage into the grid [12], and the state-of-the-art day-ahead wind forecast error is still quite significant [7], [16]. Aggregation of wind power sources at different geographical locations, on the other hand, can be an effective approach for uncertainty reduction if the wind power sources and/or their forecast errors are statistically diverse. Unlike storage, aggregation does not require significant capital investment. Thus it is expected that aggregation can complement other options of uncertainty reduction and play an important role in the process of renewable integration.

The problem of optimal contract offering in the setup where WPPs participate in forward markets has been the subject of a number of studies. Among work devising computational approaches to identify optimal forward contracts for WPPs, [17] and [18] develop stochastic programming based methods for settings with two and three successive markets, respectively. Analytical solutions to the optimal contract offering problem are derived in [19] and [20] for a perfectly competitive two-settlement market. The physical model of generation from aggregated wind turbines and small wind farms are studied in [21],

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[22], and references therein. The economic benefit of wind aggregation has been analyzed under a coalitional game formulation [23]. The design problem there is to share the total profit fairly among the WPPs in a coalition. A number of open questions are raised in [23], and will be addressed later in this paper (cf. Section IV). Other related work includes [24], which studies selling wind power with different reliability levels as an alternative method to handle uncertainty.

We propose a new instrument for wind aggregation called a *risky power contract* that allows WPPs to trade *uncertain* future power generation with each other. It enables efficient uncertainty reduction and profit maximization for every WPP via a *market mechanism*. In a day ahead market with risky power contracts as well as conventional firm power contracts, we show that the *competitive equilibrium* (CE) of the market uniquely exists, and can be computed in closed form. The CE enjoys a number of desirable properties: 1) The profit each WPP gets at the CE fairly captures its marginal contribution and diversity contribution to the whole group. 2) The CE is efficient, meaning that the total of the profits of all the WPPs equals that which would be earned by forming a grand coalition. 3) The market with risky power trades cannot be gamed, and the CE is stable. Furthermore, the CE shows that an efficient and fair profit allocation among WPPs for the coalitional game setting can be achieved in a *non-cooperative* risky power market. We evaluate the benefits of this risky power market with real world wind data (forecast and realization) from ten WPPs in the PJM Interconnection, and the locational marginal prices (day ahead and real time) from the locations of the selected WPPs. Significant benefits are observed with these ten WPPs trading risky power.

While we have presented preliminary work on risky power contracts in [1], this work addresses a much more general problem, presents different solution concepts, and provides much deeper and more comprehensive results. In particular: 1) This work studies markets with an arbitrary number of WPPs, while [1] studies two WPPs; 2) The main results of this work (cf. Theorem 1 and 2, Corollary 2 and 3) apply to arbitrary wind forecast distributions, while [1] focuses on Gaussianly distributed wind forecast; 3) This work takes into account the dependence of prices in real time power markets on wind power realization, while [1] makes a simplifying assumption that real time price is independent of wind power; 4) This work develops the competitive equilibrium in a market, while [1] develops a different and more restrictive equilibrium concept; 5) This work addresses a coalitional game using the developed results of CE, which is not considered in [1]; and 6) This work provides extensive numerical studies of the proposed risky power market using real world wind and price data, which are absent in [1].

The remainder of the paper is organized as follows. In Section II, we establish the market and wind model, define the risky power contract, and introduce the risky power market. In Section III, we provide a complete characterization of the competitive equilibrium of the risky power market, and discuss its properties and implications. In Section IV, we show that the developed CE resolves a number of issues arising in a coalitional game for profit allocation among WPPs. In Section V, numerical experiments are conducted using real world wind and price data. Conclusions are drawn in Section VI.

II. System Model

We consider N independently owned wind farms that participate in the conventional wholesale power market in order to maximize their expected profits.

A. Conventional Firm Power Market

We consider a two-settlement market system consisting of a day-ahead (DA) market and a real-time (RT) market. In the DA market, WPP i $(1 \leq i \leq N)$ offers a *firm* power forward contract, denoted by s_i, to be delivered at a future operating hour. We consider the N WPPs as price takers in the DA firm power market, and denote the price for each unit of power by $p^f \in \mathbb{R}_+$. At the operating hour, we denote the within-hour time average wind power generation of WPP i by W_i, and let $W = [W_1, \ldots, W_N]^T$. We use the following settlement procedure to model the penalty (and reward) for negative (and positive) imbalance between the wind generation at the operating hour and the DA firm power contracts.

For each unit of negative imbalance $(s_i - W_i)_+$, i.e., for the shortfall in delivering the DA-committed firm power, WPP i suffers a $\kappa \in \mathbb{R}_+$ cost or penalty. For example, $\kappa \in \mathbb{R}_+$ can be determined by the price of buying power in the RT market. For each unit of positive imbalance $(W_i - s_i)_+$, in case the surplus may be sold in the RT market or stored for future usage, WPP i gains a reward of $\lambda \in \mathbb{R}$. We note that, in practice, there are intra-hour fluctuations of wind power generation, and the ensuing imbalances are typically resolved with more than two settlement procedures including sub-hourly markets. Nonetheless, we consider the cost (or reward) due to sub-hour wind power fluctuation to be decoupled from that due to hourly mean deviations $(s_i - W_i)$. Thus, we approximate the participation of WPPs in multi-settlement power markets by the above two-settlement model, and focus on each WPP i's decision on s_i based on the DA belief of its hourly wind power average W_i. Explicit inclusion of sub-hourly markets into the model is left as future work.

When committing forward contracts in the DA market, WPPs do not know their wind power generation at the future operating hour with certainty. Such inherent uncertainty of wind power is due to the fact that wind is difficult to forecast accurately at this time scale. Thus, we model future wind generation as random variables based on whatever available forecasting mechanism. We denote the joint cumulative distribution function (cdf) of $W_1, \ldots, W_N$ by $F(w)$. The marginal cdf of W_i is denoted by $F_i(w)$. We further denote the marginal quantile function for W_i by

$$F_i^{-1}(\delta) = \inf\{w \in [0, \overline{W_i}] : \delta \leq F(w)\} \tag{1}$$

where $\overline{W}_i$ is the nameplate capacity of the wind farm i.

One day ahead, WPPs typically do not know the penalty and reward rate (κ and λ) at the future operating hour with certainty either. We employ the following *wind-realization-dependent* model on DA beliefs of κ and λ

$$\kappa = \kappa(\mathbf{1}^T \boldsymbol{W}),\ \lambda = \lambda(\mathbf{1}^T \boldsymbol{W}) \tag{2}$$

where $\mathbf{1} \in \mathbb{R}^{N \times 1}$ is the all-one vector, and $\kappa(\mathbf{1}^T \boldsymbol{W})$ and $\lambda(\mathbf{1}^T \boldsymbol{W})$ are deterministic functions of the *total wind generation* $\mathbf{1}^T \boldsymbol{W}$ among the N WPPs. Note that, one day ahead,

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$\mathbf{1}^T\boldsymbol{W}$ is a random variable, and so are κ and λ. In particular, we model $\kappa(\cdot)$ and $\lambda(\cdot)$ to be *non-increasing* functions based on the following intuition: with higher realization of total wind power (which has zero variable cost), the price in the RT power market would fall. We further model that $\kappa(W) \geq \lambda(W), \forall W$, as the price of selling power in the RT market is no higher than that of buying power (which is a necessary condition to avoid arbitrage). We note that, with inclusion of other random factors *independent* of $W_1, \ldots, W_N$ that contribute to the uncertain RT power market price, using the above model is not restrictive for the WPPs to maximize their expected profits.

B. Risky Power Contract

Because of the uncertainty of future wind power, each WPP is concerned with the deviation of its real-time wind power generation from its DA forward contract. When a WPP knows of the presence of other WPPs, it has an incentive to exploit the *statistical diversity* among different WPPs' generation by possibly mixing its own uncertain future generation with that of the others. The intuition is that, by appropriately aggregating power generation from different sources, the uncertainty in the mixture of power generation can be reduced.

For each WPP to implement this idea of aggregation in a flexible and distributed manner, we now introduce a new instrument called a *risky power contract*. The idea is to allow WPPs to trade each others' random power generation as *random* commodities (as opposed to the conventional firm power commodity). Specifically, consider WPP i buying random power from WPP j via contracts of the following form:

- One day ahead, a fraction-price pair $\{\beta_{ji}, p_{ji}\}, 0 \leq \beta_{ji} \leq 1, p_{ji} \geq 0$ is formed. Such a pair may be obtained via a bilateral contract, a bargaining process, or an auction mechanism.
- At the operating hour, W_j is revealed. WPP i receives $\beta_{ji} W_j$ from WPP j, and pays an amount $p_{ji}\beta_{ji}W_j$ to WPP j.

In other words, WPP i buys β_{ji}*-fraction* of WPP j's *random* future power W_j. At the operating hour, regardless of how much actual wind power W_j is realized, WPP i must buy $\beta_{ji}W_j$ at the price p_{ji} for each unit of power. Note that, both β_{ji} and p_{ji} are agreed upon one day ahead. The payment, in contrast, happens at the operating hour, and depends on the actual wind power realization.

Clearly, WPP i takes the risk of getting an uncertain amount of power from such a contract. Hence, we call such a pair $\{\beta_{ji}, p_{ji}\}$ a *risky* power contract. We will use risky power and uncertain power interchangeably when referring to random wind power.

C. Risky Power Market

With risky power contracts, WPPs can not only buy or sell firm power contracts in the DA market, but also trade any fractions of their uncertain future wind power $W_1, \ldots, W_N$ among each other one day ahead. $W_1, \ldots, W_N$ thus correspond to N divisible commodities in a risky power market. The motivation of introducing a risky power market is that it can lead to reduced risks for all participating WPPs that face uncertainty in their generation. Each WPP seeks a desirable *mixture* of different random power sources, in order to earn a higher profit in the conventional two-settlement firm power market.

For a WPP to optimally trade with other WPPs via risky power contracts, it needs to evaluate the benefit it gets from buying other WPPs' risky power. Clearly, such benefit depends on the *joint distribution* of the wind power (or their forecast errors) from different WPPs. To crystalize the intuition, consider WPP i buying risky power from WPP j, when the joint distribution of W_i and W_j satisfies each of the following conditions:

1) (Example of perfectly negatively correlated)
$$W_i - \mathbb{E}[W_i] = -(W_j - \mathbb{E}[W_j]).$$
2) (Example of perfectly positively correlated)
$$W_i - \mathbb{E}[W_i] = W_j - \mathbb{E}[W_j].$$
3) (Zero uncertainty) W_j is deterministic, i.e., the DA forecast of W_j is perfect.

Intuitively, in case 1), WPP i would get the most "diversity benefit" from buying and mixing W_j with W_i, as they perfectly complement each other. In cases 2) and 3), however, WPP i would not get any "diversity benefit" from buying W_j. Thus, among the three cases, WPP i would be willing to pay for W_j at the highest price in case 1), and at lower prices in case 2) and 3). Here we make the assumption that all WPPs share a common knowledge of the joint distribution of $W_1, \ldots, W_N$ one day ahead. As current DA wind power forecasting is typically based on meteorological forecasting [7], this is a reasonable assumption when meteorological forecast is openly available. The case that each WPP has private information on its own wind forecast, and can decide whether to reveal such information truthfully to other WPPs is left for future work. In practice, WPPs tend to be less correlated if they are farther apart geographically. In the mean time, aggregation cost due to, e.g., network transmission constraints tends to rise as distance between WPPs increases. In this paper, we do not incorporate network transmission constraints into the model, and the related issues are subject to future studies. We also note that, while this paper focuses on exploiting aggregation in reducing wind power uncertainty, aggregation would also help in reducing wind power variability over time. Studying the latter is left for future work, for which temporal correlations of wind power would also need to be considered (in addition to geographical correlations as considered in this paper).

With a risky power market, we are interested in precisely characterizing the benefit it brings to each participating WPP. Motivated by this, we study the competitive market equilibrium of the risky power market in the following sections.

III. Competitive Market Equilibrium

In this section, we find a stable operating point in the DA market with both firm power and risky power contracts. Such a stable operating point is termed a competitive market equilibrium, or *competitive equilibrium* (CE) in economics and game theory [25]. In particular, at a CE, each participating WPP has no incentive to deviate from its trades with other WPPs, as it already achieves its maximum expected profit. The key intuition of a CE is the following: such an equilibrium can be induced by the adoption of *a vector of prices* of the commodities in the market, which, in our setting, are the future wind power generation $W_1, \dots, W_N$. Such a vector of prices of $W_1, \dots, W_N$ then reflects how much each W_i is worth evaluated by the market.

A. Day Ahead Expected Profit of Each WPP

We assume that the variable cost of wind power production is zero. Denote the traded fractions of random power one day ahead among the N WPPs by $[\beta_{ij}] \in \mathbb{R}^{N\times N}$, where

- $\beta_{ij} \in [0,1],\ i \neq j$ denotes the fraction of W_i bought by WPP j from WPP i.
- $\beta_{ii} = 1 - \sum_{j\neq i} \beta_{ij},\ \beta_{ii} \in [0,1]$, denotes the fraction of W_i reserved by WPP i for its own use.

Accordingly, we have $[\beta_{ij}]\cdot\mathbf{1} = \mathbf{1}$. The prices of the risky power contracts are $[p_{ij}] \in \mathbb{R}^{N\times N}$. The firm power contracts of the N WPPs in the DA market are $\boldsymbol{s} = [s_1, \dots, s_N]^T$. One day ahead, the expected profit of WPP i given $[\beta_{ij}]$, $[p_{ij}]$, and $\boldsymbol{s}$ is

$$\Pi_i = p^f s_i - \sum_{j\neq i} \mathbb{E}[p_{ji}\beta_{ji}W_j] + \sum_{j\neq i} \mathbb{E}[p_{ij}\beta_{ij}W_i] \\ +\mathbb{E}\left[-\kappa(\mathbf{1}^T\boldsymbol{W})(s_i - \widetilde{W}_i)_+ + \lambda(\mathbf{1}^T\boldsymbol{W})(\widetilde{W}_i - s_i)_+\right] \quad (3)$$

where

$$\widetilde{W}_i \triangleq \sum_{j=1}^{N} \beta_{ji} W_j. \quad (4)$$

In Π_i, $p^f s_i$ is the profit earned one day ahead for selling firm power. The remaining three terms are the expected profits earned at the operating hour:

- The first expectation is the expected payment from WPP i to all other WPPs for buying fractions of their power.
- The second expectation is the expected payment WPP i receives for selling fractions of W_i to all other WPPs.
- The third expectation is the expected shortfall payment/surplus reward based on $\widetilde{W}_i$, where $\widetilde{W}_i$ is the mixture of random power available to WPP i as a result of the risky power trades.

B. Definition of Competitive Equilibrium

A competitive equilibrium in a DA market with firm and risky power contracts is defined as a set of $\boldsymbol{s}^*$, $[\beta^*_{ij}]$ and $[p^*_{ij}]$, such that the following conditions are satisfied:

1) $[\beta_{ij}]\cdot\mathbf{1} = \mathbf{1}$
2) For every WPP i, s^*_i and $[\beta^*_{ij}]$ solve the following expected profit maximization problem given $[p^*_{ij}]$:

$$\max_{s_i, [\beta_{ij}]\in\mathbb{R}_+^{N\times N}} \Pi_i\left(s_i, [\beta_{ij}], [p^*_{ij}]\right) \quad (5)$$

where Π_i is defined as in (3). Condition 1) is a *market clearing* condition: the amounts of risky power that each WPP distributes among all WPPs (including itself) must sum up to the original amount it has. Condition 2) is a *best response* condition: at the CE, the firm and risky power contracts maximize the expected profit for *every* WPP, given the prices $[p^*_{ij}]$.

Remark 1: The best response condition has strong implications as follows. Every WPP acts as if it *dictates* all the trades between itself and other WPPs. In other words, no matter what risky power trading offers $[\beta^*_{ij}]$ any WPP requests for maximizing its own profit, the other WPPs must accept them. Thus, the best response condition requires such "dictating" behavior be simultaneously true for *all* WPPs.

It is immediate to see that, at the CE, each commodity W_i $(i = 1, \dots, N)$ must have a common price to all its buyers, i.e.,

$$\forall i,\ p^*_{i1} = p^*_{i2} = \cdots = p^*_{iN} \triangleq p^*_i. \quad (6)$$

The reason is as follows. Suppose WPP j and WPP k buy $\beta^*_{ij} > 0$ and $\beta^*_{ik} > 0$ fractions of WPP i's risky power, respectively, and $p^*_{ij} < p^*_{ik}$. Then β^*_{ij} and β^*_{ik} surely do not satisfy the best response condition of WPP i, because WPP i can sell $(\beta^*_{ij} + \beta^*_{ik})$-fraction to WPP k and nothing to WPP j, with a higher expected profit. Accordingly, we can denote the prices at the CE more concisely as $\boldsymbol{p}^* = [p^*_1, \dots, p^*_N]^T$. We call $\boldsymbol{p}^*, [\beta^*_{ij}]$ and $\boldsymbol{s}^*$ the *competitive prices, competitive risky power contracts*, and *competitive firm power contracts*, respectively. We call the expected profits achieved at the CE for the N WPPs, $\Pi_i\left(s^*_i, [\beta^*_{ij}], \boldsymbol{p}^*\right)$, the *competitive payoffs*.

Clearly, CE is a stable operating point of the DA market with firm and risky power contracts. Conversely, if an operating point violates either condition of CE, it cannot be stable. This is because either the market does not clear, or there exists some WPP that seeks to change its contracts for a higher profit.

C. Results From the Single Wind Farm Case

We first study the case of only one WPP in the market, i.e., $N = 1$. In this case, there is no trading of risky power contracts among WPPs. The decision variable for the single WPP reduces to simply the firm power contract s. The best response condition (5) reduces to solving an optimal contract problem:

$$\max_{s}\ p^f s + \mathbb{E}\left[-\kappa(W)(s - W)_+ + \lambda(W)(W - s)_+\right]. \quad (7)$$

Observe that, with $\kappa(W) \geq \lambda(W), \forall W$, the objective in (7) is a concave function of s given any fixed W. Thus, (7) is a scalar convex optimization, and can be efficiently solved using, e.g., bi-section. Furthermore, the optimization problem (7) satisfies a positive homogeneity property as follows.

1) Positive Homogeneity: Given that κ and λ depend on W via (2) with $\mathbf{1}^T\boldsymbol{W} = W$, consider that someone owns b-fraction $(b > 0)$ of W, namely, bW. We define $\Pi^*(b)$ to be the maximum expected profit achievable by participating in the DA firm power market based on just bW. We have the following lemma.

Lemma 1 (Positive Homogeneity):

$$\Pi^*(b) = b\Pi^*(1), \forall b > 0. \quad (8)$$

The proof is relegated to Appendix A. In [23], this property was shown for the case that κ and λ are constants, or independent of W.

2) Closed Form Solutions for Special Cases: We now recall some existing results on closed form solutions of (7). For the case that κ and λ are constants, with $\kappa > p^f > \lambda$, the following optimal solution of (7) is found in [19]:

$$s^* = F_W^{-1}(\gamma) \quad (9)$$

where $\gamma = (p^f - \lambda)/(\kappa - \lambda) \in (0,1)$, and F_W^{-1} is the quantile function for the random variable W, as defined in (1).

With a further assumption of Gaussianly distributed wind forecast $W \sim N(\mu, \sigma^2)$, an explicit expression for $\Pi^*(1)$ is found in [1]:

$$\Pi^*(1) = p^f \mu - q\sigma \tag{10}$$

where

$$q = (\kappa - \lambda)\phi(\Phi^{-1}(\gamma)) > 0 \tag{11}$$

and $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function (pdf) and the cdf of the standard normal distribution, respectively. Equation (10) characterizes the *cost of uncertainty* of wind power for participating in the DA firm power market: the value of the uncertain power W equals that of the firm power with the same expectation μ, minus a cost term that is linear in the standard deviation of W.

D. Closed Form Solutions of Competitive Equilibrium

1) General Case: We now address the case of N WPPs. We begin with the following subproblem: *given* any weighted mixture of $W_1, \ldots, W_N$, specified by

$$\widetilde{W} = \sum_{j=1}^{N} b_j W_j \tag{12}$$

for some $b_j \geq 0, j = 1 \ldots, N$, solve the optimal firm power contract based on this $\widetilde{W}$ as follows:

$$\max_s \; p^f s + \mathbb{E}\left[-\kappa(\mathbf{1}^T \boldsymbol{W})(s - \widetilde{W})_+ + \lambda(\mathbf{1}^T \boldsymbol{W})(\widetilde{W} - s)_+\right]. \tag{13}$$

Note that κ and λ are functions of $\mathbf{1}^T\boldsymbol{W}$ instead of $\widetilde{W}$. Equation (13) can be solved in a similar fashion to our development for the single WPP case in Section III-C. Generalizing the definition of $\Pi^*(b),\ b \in \mathbb{R}_+$, we define $\Pi^*(\boldsymbol{b}),\ \boldsymbol{b} = [b_1, \ldots, b_N]^T \in \mathbb{R}_+^N$ to be the optimal value of (13) given the vector of fractions $\boldsymbol{b}$. It is immediate to generalize the positive homogeneity property (cf. Lemma 1) as follows:

Lemma 2: $\forall \boldsymbol{b} \in \mathbb{R}_+^N$, $\alpha > 0$, we have

$$\Pi^*(\alpha \boldsymbol{b}) = \alpha \Pi^*(\boldsymbol{b}). \tag{14}$$

We now provide the main results on the competitive equilibrium for N wind farms as defined in Section III-B.

Theorem 1: For $\boldsymbol{W}$ of an arbitrary joint distribution, we have the following:

1) A set of competitive prices uniquely exists as follows:

$$\forall i = 1, \ldots, N,\ p_i^* = \frac{1}{\mathbb{E}W_i} \frac{\partial \Pi^*(\boldsymbol{b})}{\partial b_i}\bigg|_{\boldsymbol{b}=\mathbf{1}}. \tag{15}$$

2) Any market clearing trades $[\beta_{ij}]$ such that

$$\forall j = 1, \ldots, N,\ \beta_{1j} = \beta_{2j} = \cdots = \beta_{Nj} \tag{16}$$

is a set of competitive risky power contracts.

3) The competitive payoffs have the following forms:

$$\forall i = 1, \ldots, N,\ \Pi_i\left(s_i^*, [\beta_{ij}^*], \boldsymbol{p}^*\right) = p_i^* \mathbb{E}W_i. \tag{17}$$

The proof is relegated to Appendix B.

2) Case of Constant κ and λ With Gaussian $\boldsymbol{W}$: We now derive the competitive prices for the case of constant κ and λ with Gaussianly distributed wind forecast: $\boldsymbol{W} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. We first define a measure of how much "risk" each WPP contributes to the group of N WPPs.

Definition 1 (Risk Contribution Index): The risk contribution index of WPP i ($i = 1, \ldots, N$) is

$$r_i \triangleq \frac{\mathbf{1}^T \boldsymbol{\Sigma}_i}{\mathbf{1}^T \boldsymbol{\Sigma} \mathbf{1}} \tag{18}$$

where $\boldsymbol{\Sigma}_i$ is the i^{th} column of $\boldsymbol{\Sigma}$.

The intuition of why r_i captures how much risk WPP i contributes to the group of N WPPs is as follows. Note that

$$\sum_{i=1}^{N} \mathbf{1}^T \boldsymbol{\Sigma}_i = \mathbf{1}^T \boldsymbol{\Sigma} \mathbf{1} = \mathrm{Var}(\mathbf{1}^T \boldsymbol{W}) \tag{19}$$

where $\mathbf{1}^T \boldsymbol{W} = \sum_{i=1}^{N} W_i$ is the full aggregation of the wind power from all the WPPs. Intuitively, $\mathrm{Var}(\mathbf{1}^T \boldsymbol{W})$ measures the risk of this full aggregation of the wind power (cf. [1, Remark 3.7]). From (18) and (19), r_i gives the fraction of the total risk (i.e., $\mathrm{Var}(\mathbf{1}^T \boldsymbol{W})$) that is contributed by WPP i. We further have that

$$0 \leq r_i \leq 1,\ \sum_{i=1}^{N} r_i = 1. \tag{20}$$

We now have the following corollary giving explicit expressions of the competitive prices:

Corollary 1: If $\boldsymbol{W} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, the competitive prices are

$$\forall i = 1, \ldots, N,\ p_i^* = p^f - \frac{q}{\mu_i} r_i \sigma_{\mathcal{N}} \tag{21}$$

where $\sigma_{\mathcal{N}} \triangleq \sqrt{\mathrm{Var}(\mathbf{1}^T \boldsymbol{W})}$.

The proof follows from applying (10) to (15).

3) Discussion: We now discuss some intuitions and implications of the closed form expressions in Theorem 1 and Corollary 1. We begin with the following observation.

Remark 2 (Incentive to Participate): Because the optimal firm power contract *without* any risky power trades is always a *feasible* solution to (5), while s_i^*, $[\beta_{ij}^*]$ is the *optimal* solution

$$\forall i = 1, \ldots, N,\ \Pi_i\left(s_i^*, [\beta_{ij}^*], \boldsymbol{p}^*\right) \geq \Pi^*(\boldsymbol{e}_i) \tag{22}$$

where $\boldsymbol{e}_i$ is the elementary vector with all-zero entries but a 1 in the ith position, corresponding to WPP i with no risky power trading. Thus, every WPP is incentivized to participate in the risky power market, because it can earn no less expected profit than by not participating.

For arbitrarily distributed wind forecast, we observe the following intuition from the competitive prices $\boldsymbol{p}^*$.

Remark 3 (Marginal Contribution to the Group): From (15), the competitive price of WPP i's risky power is linear in the marginal contribution of WPP i to the whole group of WPPs in the following sense. Consider a grand coalition of the N WPPs that aggregates *all* of their power $\mathbf{1}^T\boldsymbol{W}$. Based on this full aggregation, an optimal profit of $\Pi^*(\mathbf{1})$ can be earned by participating in the DA firm power market, just as in the single WPP case. Now, if WPP i increases an infinitesimal fraction of its risky

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power W_i, the additional profit it brings to the full aggregation determines the competitive price of W_i.

For Gaussianly distributed wind forecast, the explicit expression of p_i^* shows that, the higher risk that WPP i contributes to the whole group (i.e., a higher r_i), the less W_i is valued. Furthermore, we note that p_i^* also captures the "diversity benefit" WPP i contributes to the group as follows.

Remark 4 (Diversity Contribution): In (18)

$$\mathbf{1}^T \mathbf{\Sigma}_i = \mathbb{E}\left[(W_i - \mu_i)\left(\sum_{j=1}^{N}(W_j - \mu_j)\right)\right]$$

is the cross-covariance between WPP i's risky power W_i and the full aggregation $\mathbf{1}^T\boldsymbol{W}$. The *higher* the risk contribution index r_i is, the *more correlated* WPP i is with the full aggregation of the group, and hence the *less it contributes to the group's diversity*. From (21), a higher r_i implies a lower price for WPP i, which is consistent with the intuition that less contribution to diversity should lead to lower valuation.

For the competitive risky power contracts $[\beta_{ij}^*]$ [cf. (16)], for any WPP j, we define $\beta_j^* \triangleq \beta_{ij}^*, \ \forall i$. Equation (16) then implies the following: At the CE, *each WPP j holds a β_j^*-fraction of the full aggregation* $\mathbf{1}^T\boldsymbol{W}$. Since the market clearing condition implies that $\sum_{j=1}^{N}\beta_j^* = 1$, from positive homogeneity (cf. equation (14)), we have

$$\sum_{j=1}^{N}\Pi^*(\beta_j^*\mathbf{1}) = \sum_{j=1}^{N}\beta_j^*\Pi^*(\mathbf{1}) = \Pi^*(\mathbf{1}). \qquad (23)$$

Since the payments among the N WPPs are balanced, we arrive at the following corollary on the efficiency of the CE.

Corollary 2 (Efficiency of Competitive Equilibrium): At the competitive equilibrium

$$\sum_{i=1}^{N}\Pi_i\left(s_i^*, [\beta_{ij}^*], \boldsymbol{p}^*\right) = \Pi^*(\mathbf{1}). \qquad (24)$$

From Corollary 2, the total expected profit at the CE of all the WPPs equals the optimal profit that can be earned by aggregating all their wind power together. Note that, no matter how the N WPPs might cooperate with each other, no DA firm power contracts offered by them can bring a higher total profit than $\Pi^*(\mathbf{1})$. Therefore, the CE of the market is efficient. Moreover, since each WPP i holds a β_i^*-fraction of the full aggregation $\mathbf{1}^T\boldsymbol{W}$ at the CE, its competitive *firm* power contract must also be a β_i^*-fraction of the optimal firm power contract based on the full aggregation (cf. the proof of Lemma 1):

$$s_i^* = \beta_i^* s^* \qquad (25)$$

where s^* solves (13) with $\boldsymbol{b} = \mathbf{1}$ in (12). Again, from $\sum_{i=1}^{N}\beta_i^* = 1$ and the balanced payments, at the operating hour, we have that the sum of the actual *realized* profit (as opposed to the expected profit) of the N WPPs is the same as that which would be optimally obtained based on the full aggregation. In summary, not only do the total competitive payoffs resemble what would be achieved *in expectation* based on a full aggregation, but also they are the same *for all realizations of wind power and profit*.

For the competitive payoffs, we have the following observation.

Remark 5: From (17), with the competitive prices $\boldsymbol{p}^*$, the maximum expected profit of WPP i $(i = 1, \ldots, N)$ can be achieved by simply *selling all* W_i via a risky power contract. This is true for all WPPs, and yet does not contradict the market clearing condition for the following reason. With the competitive prices, for any WPP i, as long as the risky power trades satisfy (16), it is indifferent to choosing any $\beta_i^* \in [0,1]$, including $\beta_i^* = 0$ which means to sell all of W_i via risky power contracts. Thus, as long as $\beta_1^*, \ldots, \beta_N^*$ clears the market, none of the WPPs has any incentive to deviate from these trades.

Finally, for the stability of the competitive equilibrium, the following question naturally arises: Do the competitive payoffs allow the WPPs to *game* the market? Specifically, consider a subset of WPPs, denoted by $\mathcal{S}$, who first form a subgroup to aggregate their risky power, and then join the risky power market with the other WPPs. The question is, will the competitive payoff that this subgroup receives be *higher* than the sum of the competitive payoffs that each of them would get if they join the risky power market individually? We answer this question in the following corollary whose proof is relegated to Appendix C.

Corollary 3: For any subset $\mathcal{S}$ of the N WPPs, let $W_{\mathcal{S}} = \sum_{i\in\mathcal{S}} W_i$, and let $\hat{p}_{\mathcal{S}}^*$ be the competitive price of $\mathcal{S}$ if this subset joins a risky power market with the other WPPs. Then

$$\hat{p}_{\mathcal{S}}^*\mathbb{E}W_{\mathcal{S}} = \sum_{i\in\mathcal{S}} p_i^*\mathbb{E}W_i. \qquad (26)$$

As a result, for any subset of WPPs, forming a subgroup before joining the risky power market does not give them a higher competitive payoff. Instead, they receive exactly the *same total competitive payoff* as they would by joining the risky power market individually. Therefore, the risky power market cannot be gamed in this way.

IV. Competitive Equilibrium as a Profit Allocation Scheme in a Coalition of Wind Farms

In the last section we saw that the competitive equilibrium in a risky power market achieves efficiency: the total payoffs of the N WPPs at the CE equals the amount achieved with a full aggregation of all the WPPs, not only in expectation, but also for all realized wind power and profits. Thus, the developed competitive payoffs can be viewed as a *profit allocation scheme for a grand coalition of the N wind farms*. The intuition behind this is that the competitive payoffs provide an *evaluation of the contribution* of each WPP to the grand coalition in terms of diversity and risk-reduction, and allocate the profit accordingly.

A. Formulation of a Coalitional Game

The problem of profit allocation for a coalition of WPPs falls into a class of game theoretic problems called *coalitional games*, and has been studied in [23]. Specifically, the following provides a well-defined coalitional game of the WPPs [23]:

- A set of N WPPs, denoted by $\mathcal{N} = \{1, \ldots, N\}$.
- A function v defined over every subset $\mathcal{S} \subseteq \mathcal{N}$ as follows:

$$v(\mathcal{S}) = \Pi^*(\sum_{i\in\mathcal{S}}\boldsymbol{e}_i). \qquad (27)$$

In other words, for any subset of WPPs $\mathcal{S}$, $v(\mathcal{S})$ denotes the maximum expected profit achievable by participating in the DA firm power market based on the wind power aggregation within $\mathcal{S}$, i.e., $\sum_{i\in\mathcal{S}} W_i$. A vector $\boldsymbol{x} \in \mathbb{R}^{N\times 1}$ is a *payoff vector* if it satisfies the following:

$$\sum_{i=1}^{N} x_i = v(\mathcal{N}). \tag{28}$$

In other words, $\boldsymbol{x}$ denotes an allocation to each of the N WPPs of the maximum expected profit achievable with a full aggregation of them. A central question in coalitional games is thus, what profit allocation is stable and can be accepted by all the WPPs. The concept of *core* provides an answer to this question, and is defined as follows. A payoff vector $\boldsymbol{x}$ is said to be *in the core* of the coalitional game if it satisfies the following:

$$\forall \mathcal{S} \subseteq \mathcal{N},\ \sum_{i\in\mathcal{S}} x_i \geq v(\mathcal{S}). \tag{29}$$

In other words, under the expected profit assignment $\boldsymbol{x}$, no subset $\mathcal{S}$ of the N WPPs have an incentive to deviate from the grand coalition, as they cannot earn a higher expected profit on their own based on $\sum_{i\in\mathcal{S}} W_i$. Thus, a payoff vector in the core provides a profit allocation among the N WPPs with a stability in the above sense.

The above coalitional game [cf. (27)] is studied in [23]. It has been shown that, for the case that κ and λ are constants/ independent of $\boldsymbol{W}$, this coalitional game has a nonempty core. Since the core is defined by $2^N - 1$ linear constraints [cf. (28) and (29)], to find a payoff vector in the core, it suffices to solve a linear program with $2^N - 1$ constraints.

However, there are a number of issues related to finding a good payoff vector in the core, and to how *actual* total profit is allocated after wind power is realized. First, when the number of wind farms N is large, solving such linear programs might not be computationally efficient as the number of constraints grows exponentially with N. Next, as pointed out in [23], such solutions from solving linear programs lack a clear interpretation of *how the profit allocation is related to the correlation structure* of the WPPs. Moreover, by definition of the coalitional game (27), obtaining a payoff vector $\boldsymbol{x}$ in the core only determines how *expected* profit is allocated. It is not clear how to fairly translate a payoff vector to allocation of actual *realized* profit in the operating hour when the wind power is realized.

B. Profit Allocation Induced by Competitive Equilibrium

We now consider the general case that κ and λ depend on $\boldsymbol{W}$ [cf. (2)], and address the above issues. We have the following theorem whose proof is relegated to Appendix D.

Theorem 2: The coalitional game (27) has an non-empty core, and the vector of competitive payoffs $[p_1^*\mathbb{E}[W_1], \ldots, p_N^*\mathbb{E}[W_N]]^T$ is in the core.

Thus, the vector of competitive payoffs [cf. (17), (15), and (21)] enjoys the following properties:

- It provides in closed form a unique payoff vector in the core, and is easily computable.
- It has a clear interpretation of how the correlation structure of the N WPPs should be taken into account in profit allocation (cf. Remark 4).

Moreover, after the wind power is realized at the operating hour, the competitive firm and risky power contracts directly determine how much profit each WPP actually receives, as specified by the payment settlement procedure. In particular, as the payments for risky power contracts *depend on the wind power realization*, so does the proportion of the realized total profit that each WPP is allocated. For example, if WPP i accidentally has zero wind realized, even if it sold all of its own risky power one day ahead, it would still get zero payment from these contracts. Thus, with risky power contracts, the allocation of the realized total profit at the CE fairly captures the differences in wind power realization at different WPPs.

Finally, we note that the induced profit allocation at the CE is obtained as an equilibrium in a *non-cooperative market* enabled by risky power contracts, as opposed to being computed from a cooperative game setting. Thus, the developed risky power contract has the potential to lead to fair and efficient profit allocation in a competitive market. The design of the specific bargaining/auction rules and the dynamics of the risky power market are left for future work.

V. Numerical Experiments

A. Data Description and Preparation

We obtained wind power forecast and actual generation data of ten WPPs from the NREL dataset [26]. Ten WPPs within an area of 75 miles radius, which is reasonably close to have low aggregation cost (e.g., transmission cost), and yet sufficiently far for getting statistically diverse wind forecast errors, are selected. For WPP i, $i = 1, \ldots, 10$, and hour t, we use the model

$$W_i(t) = \widehat{W}_i(t) + \epsilon_i(t)$$

where $\widehat{W}_i(t)$ is the forecast wind power generation and $\epsilon_i(t)$ is the wind power forecast error. For numerical simplicity, we assume $(\epsilon_1(t), \ldots, \epsilon_{10}(t))$ are independent and identically distributed (i.i.d.) across time t, and

$$((\epsilon_1(t), \ldots, \epsilon_{10}(t)) \sim N(\mathbf{0}, \widehat{\Sigma}) \tag{30}$$

where $\mathbf{0} \in \mathbb{R}^{10\times 1}$ and $\widehat{\Sigma}$ is estimated using wind power forecast error data in January 2004. Here our goal is to demonstrate the usefulness of the proposed risky power trading, even when the joint distribution of wind power forecast error (and thus wind power generation) is estimated using such a simple model. Sophisticated statistical models (e.g., models considering periodicities due to time of the day and/or seasonality, or models using non-parametric estimation of the joint distribution) can be adopted here, and it is expected that better modeling and estimation of the joint wind power generation can further improve the benefit of aggregation via risky power trading.

These ten wind farms are located in the PJM interconnection, whose locational marginal prices in DA, denoted by $p^{\mathrm{DA}}(t)$ [which corresponds to $p^f(t)$ in our model], and RT, denoted by $p^{\mathrm{RT}}(t)$, for each hour t in February 2004 have been obtained for simulation. Notice that trading decisions happen in DA markets, when the RT prices are unknown to WPPs. To avoid large RT penalties (due to limited RT market volume and large RT price volatility), WPPs are well motivated to be conservative in the following sense: even if they

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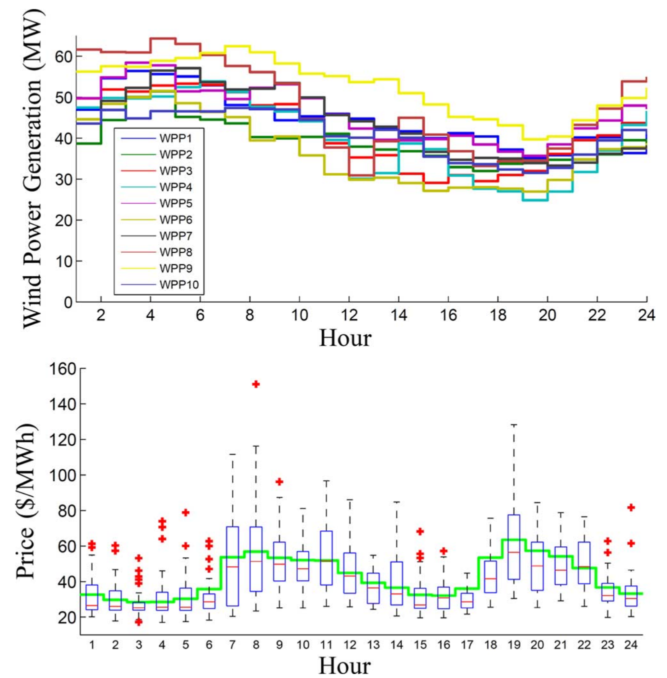


Fig. 1. Wind power generation (above) and price (below) data profiles. The wind data and DA price data (green solid line below) are averaged across days of the month. Box plots are used to illustrate the volatility of RT prices.

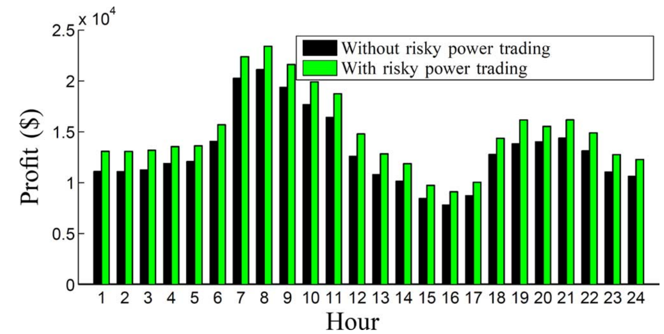


Fig. 2. Comparison of the total profits earned in different scenarios. All data are averaged across days of the month.

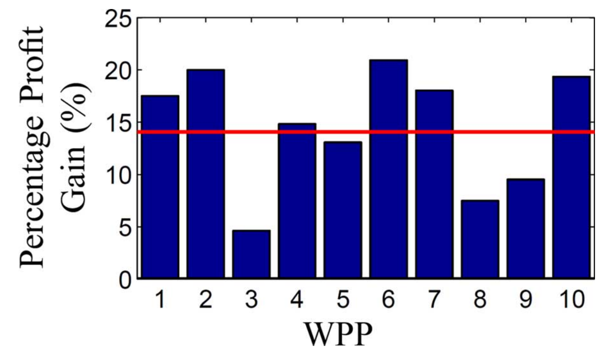


Fig. 3. Percentage profit gain of each WPP by enabling risky power trading. The red line shows the percentage profit increase of the total profit.

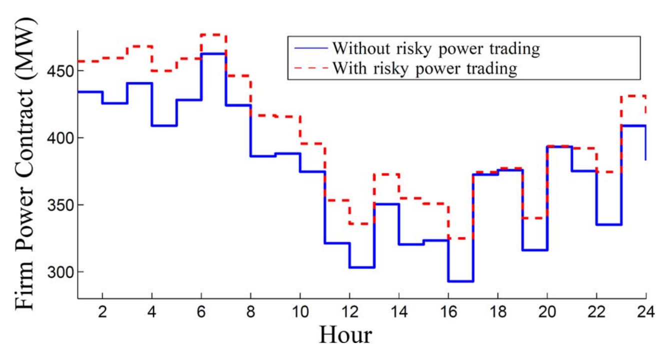


Fig. 4. Comparison of the total firm power contract levels in different scenarios. All data are averaged across days of the month.

could forecast the RT prices accurately in expectation, they are incentivized to act as if they see a lower payment rate for positive imbalance and higher payment rate for negative imbalance. In view of this, we generate the $\kappa(t)$ and $\lambda(t)$ sequences such that $\kappa(t) = \max(1.2p^{\mathrm{DA}}(t), 2p^{\mathrm{RT}}(t))$, and $\lambda(t) = \min(p^{\mathrm{DA}}(t)/1.2, p^{\mathrm{RT}}(t)/2)$.

We note that, as we base our simulation on the real world RT price data, the wind-realization-dependent price model (2) is not invoked here. Hence there is not a specific wind penetration level associated with our simulation.

All numerical results are produced using Matlab 2013a on a laptop with an Intel Core i5 1.3-GHz CPU with 4 GB of RAM.

B. Simulation and Results

We simulate the bidding process using the wind power and price data in February 2004 (shown in Fig. 1), in the following two scenarios. In the first scenario, for each hour t, each WPP sells a firm power contract separately. This is the baseline scenario without risky power trading. In the second scenario, for each hour t, in addition to selling firm power contracts, WPPs also trade risky power contracts with each other, where we assume the risky power market is at the competitive equilibrium. The simulation for these 696 hours is done with a running time 0.56 seconds. The total profits of all WPPs, averaged across days in the month, for both scenarios are depicted in Fig. 2. It is evident that in all hours the average total profits of the ten WPPs with risky power trading are higher than that without risky power trading. The total profit summed over all hours and WPPs is increased by 14.08%. The profit gain for each WPP is depicted in Fig. 3. Among the 696 simulated hours, the scenario with risky power trading has a higher total profit of the ten WPPs in 657 hours (i.e., 94.40% of the simulation period). The total realized profit with risky power trading can at times be lower than the realized profit without risky trading due to unfavorable realization of wind forecast errors, or inaccuracy in the statistical model of the joint wind distribution. Nevertheless, it is shown here that both on average and with high probability, risky power trading improves the total profit.

Fig. 4 shows the total firm power contracts of the ten WPPs with and without risky power trading. Higher total firm power contracts are observed consistently in the case with risky power trading. This is because the aggregation effect enabled by risky power trading reduces the uncertainty of all WPPs, and hence encourages higher DA firm power contracts.

Finally, we consider a setting with the number of wind farms varying from 1 to 10. The idea is to illustrate how the performance of aggregation via risky power trading improves as the number of participating WPPs increases. One empirical

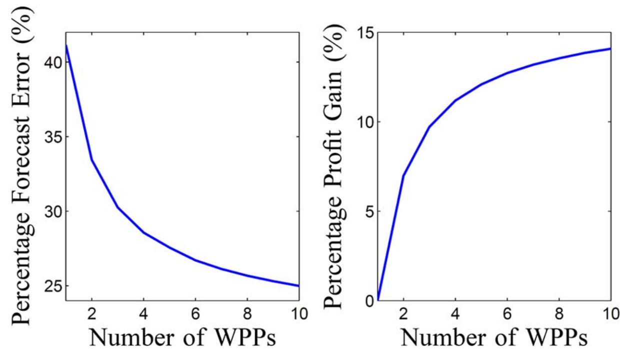


Fig. 5. Percentage forecast error (left panel) and percentage profit gain due to aggregation (right panel) with increasing numbers of WPPs.

justification of this performance improvement is that the percentage forecast error of an aggregation of WPPs decreases as the number of WPPs increases. Fig. 5 (left panel) shows how the percentage forecast errors (measured in the sum of absolute error sense) change with the size of aggregation using the data of the ten WPPs. Here, the forecast of the aggregation is computed by simply summing up the individual forecast. We note that a joint forecast of the aggregation can potentially perform even better in reducing the forecast error of the aggregation. To average out the effect of the order of aggregation, i.e., which WPP gets into the aggregation first, second, and so on, we simulate the process with 500 random orders and report the average results. Fig. 5 (right panel) depicts the percentage total profit gains for different aggregation sizes. It is worth noting that, significant profit gain can be achieved with aggregation of just a few WPPs.

VI. Conclusions

We have proposed risky power contracts for WPPs to trade uncertain future wind power in a noncooperative market, so that a reduced risk and an increased profit from wind aggregation are achieved for every WPP. We have shown that, in a two-settlement market with risky and firm power trading, a competitive equilibrium of the market uniquely exists, and is easily computable in closed form. The competitive payoff of each WPP captures both the marginal contribution and the diversity contribution provided by this WPP to the whole group of WPPs. The CE has been shown to be efficient: at the CE, the total payoff among the WPPs equals the amount achieved by a grand coalition, not only in expectation one day ahead, but also for all realized profits at the operating hour. We have shown that the risky power market cannot be gamed by WPPs who form subgroups before joining the market, and the CE is stable. In a coalitional game setting, the profit allocation induced by the CE has been shown to be always in the core of the game. We have evaluated the benefits of the risky power market based on wind data (including forecasts and realizations) of ten WPPs in the PJM interconnection, and locational marginal price data from the locations of the ten WPPs. Even using a very simple model on DA estimation of the joint wind power distribution and the real time prices, with these ten WPPs trading risky power, we have observed a close to 15% increase in the realized profit for the ten WPPs.

Appendix A
Proof of Lemma 1

Proof: 1) We prove that $\Pi^*(b) \geq b\Pi^*(1)$:

For $b = 1$, denote the optimal solution that achieves Π^* (1) by s^*. For any $b > 0$, we let $s = bs^*$. For any realization of W, the achieved profit based on bW with $s = bs^*$ equals b times the achieved profit based on W with $s = s^*$. Thus, the same equality holds in expectation, and $b\Pi^*$ (1) is always achievable based on bW.

2) We prove that $\Pi^*(1) \geq \Pi^*(b)/b$:

For $b > 0$, denote the optimal solution that achieves $\Pi^*(b)$ by s_b^*. For $b = 1$, we let $s = s_b^*/b$. For any realization of W, the achieved profit based on W with $s = s_b^*/b$ equals $1/b$ times the achieved profit based on bW with $s = s_b^*$. Thus, the same equality holds in expectation, and $\Pi^*(b)/b$ is always achievable based on W. ■

Appendix B
Proof of Theorem 1

Before we prove Theorem 1, we first prove the following lemma.

Lemma 3: $\Pi^*(\boldsymbol{b})$ is a nondecreasing concave function of $b_j,\ b_j \geq 0, \forall j$, where $\boldsymbol{b} = [b_1 \ldots b_N]^T$.

Proof: The nondecreasing property follows from the fact that having additional wind power aggregated can never decrease the optimal expected profit. For concavity, we have that $\forall \alpha \in [0,1],\ \boldsymbol{b}, \tilde{\boldsymbol{b}} \in \mathbb{R}_+^N$

$$\begin{aligned}
&\Pi^*\left(\alpha \boldsymbol{b} + (1-\alpha)\tilde{\boldsymbol{b}}\right)\\
&\geq \Pi^*\left(\alpha \boldsymbol{b}\right) + \Pi^*\left((1-\alpha)\tilde{\boldsymbol{b}}\right) \qquad (31)\\
&= \alpha\Pi^*(\boldsymbol{b}) + (1-\alpha)\Pi^*(\tilde{\boldsymbol{b}}). \qquad (32)
\end{aligned}$$

Equation (31) is because the sum of the optimal expected profits based on $\alpha\boldsymbol{b}^T\boldsymbol{W}$ and $(1-\alpha)\tilde{\boldsymbol{b}}^T\boldsymbol{W}$, respectively, can surely be achieved if $\alpha\boldsymbol{b}^T\boldsymbol{W}$ and $(1-\alpha)\tilde{\boldsymbol{b}}^T\boldsymbol{W}$ are aggregated together. Equation (32) is from positive homogeneity [cf. (14)]. ■

Proof of Theorem 1: We first reduce the computation of the competitive equilibrium with both firm and risky power contracts to that with risky power contracts only. From (5)

$$\begin{aligned}
&\max_{s_i, [\beta_{ij}] \in \mathbb{R}_+^{N\times N}} \Pi_i\left(s_i, [\beta_{ij}], [p_{ij}^*]\right)\\
&\Leftrightarrow \max_{[\beta_{ij}] \in \mathbb{R}_+^{N\times N}} \max_{s_i} \Pi_i\left(s_i, [\beta_{ij}], [p_{ij}^*]\right). \qquad (33)
\end{aligned}$$

We then have the optimal expected profit for WPP i *given* risky power contracts $[\beta_{ij}]$ as follows:

$$\begin{aligned}
&\Pi_i\left(s_i^*\left([\beta_{ij}]\right), [\beta_{ij}], [p_{ij}^*]\right)\\
&= -\sum_{j\neq i} \mathbb{E}[p_{ji}\beta_{ji}W_j] + \sum_{j\neq i} \mathbb{E}[p_{ij}\beta_{ij}W_i] + \Pi^*\left([\beta_{1i}\ldots\beta_{Ni}]^T\right). \qquad (34)
\end{aligned}$$

From (6), we let $p_{j1} = \cdots = p_{jN} \triangleq p_j,\ \forall j$. Further note that $\sum_{j\neq i}\beta_{ij} = 1 - \beta_{ii}$, and define $\mu_j \triangleq \mathbb{E}[W_j], \forall j$. Thus

$$(34) = -\sum_{j\neq i} p_j\mu_j\beta_{ji} + \sum_{j\neq i} p_i\mu_i\beta_{ij} + \Pi^*\left([\beta_{1i}\ldots\beta_{Ni}]^T\right),$$

$$= p_i\mu_i + \left[-\sum_j p_j\mu_j\beta_{ji} + \Pi^*\left([\beta_{1i}\ldots\beta_{Ni}]^T\right) \right]. \quad (35)$$

For WPP i to satisfy the best response condition (33), WPP i solves (35) with the decision variables $\beta_{ji}, j = 1,\ldots,N$.

We make the following key observation: The N WPPs $i = 1,\ldots,N$ all solve the *same optimization problem* as follows:

$$\max_{\boldsymbol{b}\geq 0} f(\boldsymbol{b}), \text{ where } f(\boldsymbol{b}) \triangleq -\sum_j p_j\mu_j b_j + \Pi^*(\boldsymbol{b}). \quad (36)$$

We have replaced β_{ji} in (35) by b_j, and left out the constant term $p_i\mu_i$.

For the remainder of the proof, we first develop two necessary conditions for any CE if it exists. These necessary conditions then lead to $\boldsymbol{p}^*$ and $[\beta^*_{ij}]$ specified in (15) and (16), which indeed give a CE.

1) If a CE Exists, Then With Any $\boldsymbol{p}^*$ in a CE, $\mathbf{0} \in \mathbb{R}^n$ Must Be an Optimal Solution of (36): Assume a CE exists, and let $\boldsymbol{p}^*$ and $[\beta^*_{ij}]$ be in a CE. Suppose $\mathbf{0}$ is *not* an optimal solution of (36) given $\boldsymbol{p}^*$.

Let $\tilde{\boldsymbol{b}} = [\beta^*_{11}\ldots\beta^*_{N1}]^T$. From the best response condition of CE, $\tilde{\boldsymbol{b}}$ must be an optimal solution of (36). Now, $\forall\alpha \geq 0$, from positive homogeneity [cf. (14)], $\Pi^*(\alpha\tilde{\boldsymbol{b}}) = \alpha\Pi^*(\tilde{\boldsymbol{b}})$. Since the remaining part of (36) is linear in $\boldsymbol{b}$, we have $f(\alpha\tilde{\boldsymbol{b}}) = \alpha f(\tilde{\boldsymbol{b}})$. When $\alpha = 0$, as $\mathbf{0}$ is not an optimal solution of (36), $f(\mathbf{0}) = 0 < f(\tilde{\boldsymbol{b}})$. Thus, by letting α be arbitrarily large, $f(\alpha\tilde{\boldsymbol{b}})$ can also be arbitrarily large. This contradicts $\tilde{\boldsymbol{b}}$ being an optimal solution of (36).

2) If a CE Exists, Then With Any $\boldsymbol{p}^*$ in a CE, $\mathbf{1} \in \mathbb{R}^n$ Must Be an Optimal Solution of (36): Assume a CE exists, and let $\boldsymbol{p}^*$ and $[\beta^*_{ij}]$ be in a CE. Suppose $\mathbf{1}$ is *not* an optimal solution of (36) given $\boldsymbol{p}^*$.

From Lemma 3, (36) is a convex optimization, and hence the set of optimal solutions is a convex set. Since a CE must satisfy the market clearing condition, $[\beta^*_{ij}]$ cannot be an all-zero matrix. Thus (36) must have an optimal solution not equal to $\mathbf{0}$. Meanwhile, we have shown that $\mathbf{0}$ must be an optimal solution, and $\forall\tilde{\boldsymbol{b}} \neq \mathbf{0}$ in the optimal solution set, $\alpha\tilde{\boldsymbol{b}}, \forall\alpha \in \mathbb{R}_+$ is also an optimal solution. Therefore, the optimal solution set of (36) must be a *cone* [27]. Note that, the market clearing condition of the CE implies that there are N optimal solutions of (36) such that *their sum equals to* $\mathbf{1}$. This further implies that $\mathbf{1}$ is in the convex cone of optimal solutions, and contradicts that $\mathbf{1}$ is not an optimal solution.

Finally, applying the first order conditions of optimality to (36) at $\boldsymbol{b} = \mathbf{1}$ leads to $\boldsymbol{p}^*$ and $[\beta^*_{ij}]$ specified in (15) and (16). It is immediate to check that they indeed give a CE, and (17) holds. ■

APPENDIX C
PROOF OF COROLLARY 3

Proof: From (15), it suffices to prove that

$$\sum_{i\in\mathcal{S}} \frac{\partial\Pi^*(\boldsymbol{b})}{\partial b_i}\bigg|_{b_i=1,\forall i} = \frac{\partial\Pi^*\left(\sum_{i\in\mathcal{S}}\alpha\boldsymbol{e}_i + \sum_{i\notin\mathcal{S}} b_i\boldsymbol{e}_i\right)}{\partial\alpha}\bigg|_{\alpha=1,b_i=1,\forall i\notin\mathcal{S}} \quad (37)$$

which follows directly from the chain rule. ■

APPENDIX D
PROOF OF THEOREM 2

Proof: It suffices to recognize the risky power market as a *market with transferable payoff* [25], in which the utility functions of the WPPs have been proven in Lemma 3 to be nondecreasing and concave. We then invoke [25, Proposition 267.1]. ■

REFERENCES

[1] Y. Zhao, J. Qin, R. Rajagopal, A. Goldsmith, and H. V. Poor, "Risky power forward contracts for wind aggregation," in *Proc. Allerton Conf. Commun., Control and Computing*, 2013.

[2] North American Electric Reliability Corporation, "Accommodating high levels of variable generation," Tech. Rep., 2009.

[3] H. Holttinen, P. Meibom, A. Orths, B. Lange, M. O'Malley, J. O. Tande, A. Estanqueiro, E. Gomez, L. Soder, G. Strbac, J. C. Smith, and F. v. Hulle, "Impacts of large amounts of wind power on design and operation of power systems, results of IEA collaboration," *Wind Energy*, vol. 14, no. 2, pp. 179–192, 2011.

[4] E. Bitar and K. Poolla, "Selling wind power in electricity markets: The status today, the opportunities tomorrow," in *Proc. Amer. Control Conf.*, 2012, pp. 3144–3147.

[5] National Renewable Energy Laboratory, "Calculating wind integration costs: Separating wind energy value from integration cost impacts," Tech. Rep., 2009.

[6] Y. Makarov, C. Loutan, J. Ma, and P. d. Mello, "Operational impacts of wind generation on California power systems," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 1039–1050, May 2009.

[7] P. Pinson, "Wind energy: Forecasting challenges for its operational management," *Statist. Sci.*, vol. 28, no. 4, pp. 564–585, 2013.

[8] National Renewable Energy Laboratory, "Western wind and solar integration study," Tech. Rep., 2010.

[9] C. Klessmann, C. Nabe, and K. Burges, "Pros and cons of exposing renewables to electricity market risks—a comparison of the market integration approaches in Germany, Spain, and the UK," *Energy Policy*, vol. 36, no. 10, pp. 3646–3661, Oct. 2008.

[10] L. Bird, J. Cochran, and X. Wang, "Wind and solar energy curtailment: Experience and practices in the United States," NREL Tech. Rep., 2014.

[11] J. R. Abbad, "Electricity market participation of wind farms: the success story of the Spanish pragmatism," *Energy Policy*, vol. 38, no. 7, pp. 3174–3179, Jul. 2010.

[12] National Renewable Energy Laboratory, "The role of energy storage with renewable electricity generation," Tech. Rep., 2010.

[13] E. Bitar, R. Rajagopal, P. Khargonekar, and K. Poolla, "The role of co-located storage for wind power producers in conventional electricity markets," in *Proc. Amer. Control Conf.*, 2011, pp. 3886–3891.

[14] J. Qin, H. Su, and R. Rajagopal, "Storage in risk limiting dispatch: Control and approximation," in *Proc. Amer. Control Conf.*, 2013.

[15] R. Rajagopal, D. Tse, and B. Zhang, "Network risk limiting dispatch: Optimal control and price of uncertainty," *IEEE Trans. Automat. Control*, to be published.

[16] T. Hong, P. Pinson, and S. Fan, "Global energy forecasting competition 2012," *Int. J. Forecast.*, vol. 30, no. 2, pp. 357–363, 2014.

[17] G. Bathurst, J. Weatherill, and G. Strbac, "Trading wind generation in short term energy markets," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 782–789, Aug. 2002.

[18] J. Morales, A. Conejo, and J. Perez-Ruiz, "Short-term trading for a wind power producer," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 554–564, Feb. 2010.

[19] E. Bitar, R. Rajagopal, P. Khargonekar, K. Poolla, and P. Varaiya, "Bringing wind energy to market," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1225–1235, Aug. 2012.

[20] M. Zugno, T. Jhonsson, and P. Pinson, "Trading wind energy on the basis of probabilistic forecasts both of wind generation and of market quantities," *Wind Energy*, vol. 16, no. 6, pp. 909–926, 2013.

[21] J. Slootweg and W. Kling, "Aggregated modelling of wind parks in power system dynamics simulations," in *Proc. IEEE Bologna Power Tech Conf.*, Jun. 2003.

[22] M. Ali, I.-S. Ilie, J. Milanovic, and G. Chicco, "Wind farm model aggregation using probabilistic clustering," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 309–316, Feb. 2013.

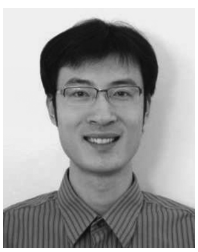
[23] E. Baeyens, E. Bitar, P. Khargonekar, and K. Poolla, "Coalitional aggregation of wind power," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3774–3784, Nov. 2013.

[24] E. Bitar, K. Poolla, P. Khargonekar, R. Rajagopal, P. Varaiya, and F. Wu, "Selling random wind," in *Proc. 45th Hawaii Int. Conf. Syst. Sci. (HICSS)*, 2012, pp. 1931–1937.

[25] M. J. Osborne and A. Rubinstein*, A Course in Game Theory*. Cambridge, MA, USA: MIT Press, 1994.

[26] National Renewable Energy Laboratory, "Eastern wind integration and transmission study," Tech. Rep., 2010.

[27] S. Boyd and L. Vandenberghe*, Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.



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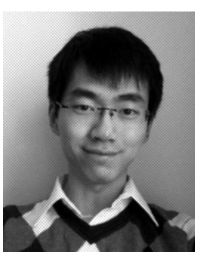

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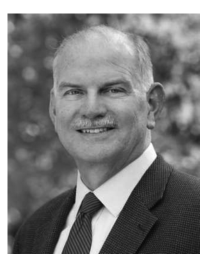

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