# **Risky Power Forward Contracts for Wind Aggregation**

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Abstract-Risky power contracts are introduced for enabling wind power aggregation. First, the problem of optimal risky and firm power contract offering in the forward market is formulated in the single wind farm setting. Analytical solutions are obtained, and the concepts of fair price of wind power and price of unitized risk are introduced. The more general setting of two wind farms both trading risky and firm power is studied, in which both wind farms seek to benefit from wind aggregation. The problem of a contract offering game in the forward market is formulated. Analytical solutions are obtained for the best responses that reveal clear insights into the optimal firm and risky contract offering for each wind farm. Complete characterization of the equilibria of the game is then obtained analytically. A generalization of the fair price to the two wind farm setting is derived, which characterizes the value of wind aggregation. With the generalized fair prices, all equilibria are also efficient, namely, they achieve the same total profit as forming a coalition of the two wind farms.

## I. INTRODUCTION

The need for integrating more wind energy into the electric grid calls for a major re-thinking in the design and operation of the power markets [1], [2], [3]. California, for example, anticipates 33% renewable penetration by 2020, within which wind energy will play a crucial role. Due to the intrinsic uncertainty and variability in wind power generation, such aggressive penetration goals can hardly be achieved with the current worst-case dispatch procedures which are designed for small uncertainty scenarios [4], [5]. Under these scenarios, operating reserves, typically supplied by *expensive fast*ramping fuel-based generators, are scheduled to compensate for forecast errors in the load, which are often as low as 1% - 3%. As wind power generation is difficult to forecast on horizons longer than fifteen minutes, significant additional reserve capacity is needed to accommodate the uncertainty brought into the system by the increasing wind penetration (see e.g. [6]). This will greatly increase the system cost and offset the environmental benefits of wind power due to greenhouse gas produced by these fast-ramping generators.

In particular, the approach of taking all wind power generation into the system as negative load via extra-market procedures such as feed-in tariffs [7] (used in Germany and many regions of the US) is not likely to work when the wind penetration level is high. For scenarios of over 30% wind penetration, the cost of increased reserve margin socialized among load serving entities (LSEs) can become excessively high, and hence discourages LSEs to accept high wind penetration. A primary alternative approach, carried out in the UK for example, requires wind power producers (WPPs) to participate in conventional electricity wholesale markets, and imposes a financial penalty on WPPs for deviations from contracts offered in the forward market [8]. Such a market structure provides a strong incentive for WPPs to *firm* their own wind power generation, that is to reduce the generation uncertainty and variability, via a range of technical and market options. Among these options, aggregation of statistically diverse wind power sources is a very effective approach.

The problem of optimal contract offering in the setup where WPPs participate in forward markets has been the subject of a number of studies. Among work devising computational approaches to identifying optimal forward contracts for WPPs, [9] and [10] develop stochastic programming based methods for settings with two and three successive markets, respectively. Analytical solutions to the optimal contract offering problem are derived in [11] and [12] for a perfectly competitive two-settlement market. Relatively less analytical work has been conducted for the problem of wind aggregation. [13] considers the problem of how to form a willing coalition among a group of WPPs based on the statistics of the wind power generation process for each wind farm. As it is easy to demonstrate the benefit of aggregation in this setting, the focus is on the design of a mechanism to share profit fairly among participating WPPs. Other related work includes [14], which studies the problem of selling wind power with different reliability levels. The work concentrates on how to allocate randomly generated wind power to different consumers who accept power supply with different reliability levels.

This paper proposes a new approach to achieve wind aggregation by allowing WPPs to trade *risky power* in the forward market. This approach has the benefit of being much more *flexible* than existing approaches in the following two aspects. (i) Instead of imposing a binary decision that requires a WPP to either fully aggregate or not with another WPP, our approach allows a WPP to acquire any portion of the wind power from another WPP. In fact, in many cases as will be shown later in this paper, there exists a nontrivial

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optimal portion for such acquisition. (ii) After introducing risky power contracts into forward markets in addition to firm (riskless) power contracts, system operators or third party aggregators are now able to flexibly use firm power and risky power in forward markets to form new risky power contracts of *any reliability level*. Thus, our proposed approach creates significant supply flexibility to fulfill interruptible power demands (e.g. power consumption of refrigerators, HVAC systems and electric vehicles) with different reliability levels [14], [15].

Under our setting of trading risky power, each WPP can achieve wind aggregation by buying risky power produced by other WPPs. We first formulate the risky and firm power contract offering problem for a single WPP (Section II). We then provide analytical solutions to this problem (Section III), and introduce the concept of fair price and price of unitized risk for the single WPP setting. Next, we generalize the setting to two WPPs where wind aggregation plays a central role (Section IV), and formulate the problem of a contract offering game. We provide analytical solutions of the optimal contract that achieves the best wind aggregation for each WPP (Section V). Based on the derived best responses, we provide analytical solutions of the equilibrium contracts for the two WPPs (Section VI), and introduce the generalized fair prices that capture the value of aggregation between the two WPPs. Section VII concludes the paper with a discussion on future work.

#### II. SINGLE WIND FARM PROBLEM

We start by introducing the problem of a single wind farm. Analytical solutions to the problem provide a theoretical basis for analyzing the case of more than one wind farm. Our single wind farm model is similar to that of [11] except that we introduce a new *risky* wind power commodity.

#### A. Wind Model

For each operating hour under consideration, the withinhour time average random wind power generation is denoted by W. We assume that the statistics of W are known, with its cumulative distribution function (cdf) being F(w). Denote the quantile function by

$$F^{-1}(\delta) = \inf\{w \in [0, \overline{W}] : \delta \le F(w)\},\$$

where  $\overline{W}$  is the nameplate capacity of the wind farm.

#### B. Market Model

Consider a two-settlement market system consisting of a day-ahead (DA) market and a real-time (RT) market. In the DA market, in addition to offering a *firm* power contract s at price  $p^{f} \in \mathbb{R}_{+}$ , the WPP can also offer a *risky* power contract  $\alpha W$ ,  $\alpha \in [0, 1]$ , at price  $p^{r} \in \mathbb{R}_{+}$ . Here, we define  $p^{r}$  to be the payment for the risky power  $\alpha W$  divided by  $\mathbb{E}(\alpha W)$ , namely, the per unit payment computed based on the *expected wind generation*. In the RT market, the wind generation W is revealed to the WPP. A settlement procedure for imbalances corresponding to the firm power contract is then applied. For each unit of negative imbalance  $(s - (1 - \alpha)W)_{+}$ , i.e., the shortfall in delivering the DA-committed

firm power, the WPP suffers a  $\kappa \in \mathbb{R}_+$  cost or penalty; for each unit of positive imbalance  $((1 - \alpha)W - s_+)$ , in case the surplus may be sold in the spot market or stored for later usage, the WPP gains a reward  $\lambda \in \mathbb{R}_+$ . Here the imbalance is computed using  $(1-\alpha)W$  as the wind generation available to fulfill the firm power contract, because the  $\alpha$ -portion of the wind power generation has been sold as a risky power contract in the DA market.

We assume that the WPP is a *price taker* for both firm and risky power in the DA market. Thus  $p^{\rm f}$  and  $p^{\rm r}$  are constants independent of *s* and  $\alpha$ . Further, we assume  $\kappa$ and  $\lambda$  to be deterministic and known to the WPP in the DA market. Note that following the development in [11], one can extend this setup to the case in which  $\kappa$  and  $\lambda$  are random but independent of the wind process. The interesting and challenging setting where  $\kappa$  and  $\lambda$  depend on the wind process through market mechanisms is left for future research. Regarding the prices, we assume (i)  $p^{\rm f} \ge p^{\rm r}$ , i.e., potential consumers of the risky power are *risk averse*, and (ii)  $\kappa > p^{\rm f} > \lambda$  to avoid arbitrage opportunity and to incentivize the WPP to sell firm power in the DA market as opposed to holding all its generation for the RT market.

# C. Optimal Contract Offering Problem

Assuming that the variable cost of wind power production is zero, the optimal DA firm and risky contract offering problem for the WPP is

$$\max_{s,\alpha} \quad \Pi(\alpha, s) \tag{1a}$$

s.t. 
$$0 \le \alpha \le 1$$
, (1b)

where the expected profit for the wind farm is

$$\Pi(\alpha, s) = p^{\mathrm{r}} \alpha \mu + p^{\mathrm{t}} s$$

$$+ \mathbb{E} \Big[ -\kappa (s - (1 - \alpha)W)_{+} + \lambda ((1 - \alpha)W - s)_{+} \Big].$$
(2)

#### III. OPTIMAL CONTRACTS FOR A SINGLE WIND FARM

### A. Optimal Contracts for General Wind Distribution

In order to solve problem (1a) analytically, we can express the optimization as

$$\max_{\alpha \in [0,1]} p^{\mathsf{r}} \alpha \mu + \max_{s} \left\{ h(s,\alpha) \right\}$$
(3)

where the objective of the inner maximization (i.e. the profit earned by selling firm power contract) is

$$h(s,\alpha)$$
  
=  $p^{\mathbf{f}}s + \mathbb{E}\Big[-\kappa(s - (1 - \alpha)W)_{+} + \lambda((1 - \alpha)W - s)_{+}\Big]$ 

The following lemma provides a closed-form solution to the inner maximization parametrized by  $\alpha$ , which in turn gives an explicit expression for the objective function of the outer maximization.

Lemma 3.1: For each  $\alpha \in [0,1]$ , denote  $s^*(\alpha) = \operatorname{argmax}_{s}\{h(s,\alpha)\}$ . The optimal firm power contract is

$$s^{\star}(\alpha) = (1 - \alpha)F^{-1}(\gamma), \tag{4}$$

and the corresponding optimal profit by selling a firm power contract is

$$h(s^{\star}(\alpha), \alpha) = (1-\alpha) \Big\{ (p^{\mathrm{f}} F^{-1}(\gamma) + \mathbb{E} \big[ -\kappa (F^{-1}(\gamma) - W)_{+} + \lambda (W - F^{-1}(\gamma))_{+} \big] \Big\}, \quad (5)$$

where  $\gamma = (p^{\rm f} - \lambda)/(\kappa - \lambda) \in (0, 1)$ .

Remark 3.2 (Opportunity cost of risky power): Eq. (5) reveals that the outer maximization in problem (3) has an objective  $\Pi(\alpha, s^*(\alpha))$  that is affine in  $\alpha$ . As the opportunity cost for the WPP to sell more risky power is the potential benefit it can gain from selling the corresponding amount of firm power, (5) further justifies the linear payment scheme (cf. (2)) adopted in this paper for risky power.

Here the marginal opportunity cost of risky power is

$$-\frac{dh(s^{\star}(\alpha),\alpha)}{d\alpha}$$
  
=  $p^{f}F^{-1}(\gamma) + \mathbb{E}\left[-\kappa(F^{-1}(\gamma)-W)_{+} + \lambda(W-F^{-1}(\gamma))_{+}\right]$ 

based upon which we can define the following critical quantity for the WPP:

Definition 3.3: The fair price  $p^{r*}$  for a WPP with given cdf F is defined such that

$$p^{r*}\mu = -\frac{dh(s^{*}(\alpha), \alpha)}{d\alpha}$$
  
=  $p^{f}F^{-1}(\gamma) + \mathbb{E}\left[-\kappa(F^{-1}(\gamma) - W)_{+} + \lambda(W - F^{-1}(\gamma))_{+}\right]$   
Notice that the fair price  $p^{r*}$  clearly depends on the distri-

bution of W. With the definition of fair price, we can state the closed-form optimal firm/risky contract offering for the wind farm:

Corollary 3.4: The optimal DA firm and risky contracts are

$$(s^{\star}, \alpha^{\star}) = \begin{cases} (F^{-1}(\gamma), 0) & \text{if } p^{r} < p^{r^{\star}}, \\ (0, 1) & \text{if } p^{r} > p^{r^{\star}}. \end{cases}$$

In the case  $p^r = p^{r\star}$ , any pair  $(s^{\star}, \alpha^{\star}) \in \{(s, \alpha) : \alpha \in [0, 1], s = (1-\alpha)F^{-1}(\gamma)\}$  are optimal contracts. The optimal expected profit is

$$\Pi^{\star} = \begin{cases} p^{\mathbf{r} \star} \mu & \text{if } 0 \le p^{\mathbf{r}} \le p^{\mathbf{r} \star}, \\ p^{\mathbf{r}} \mu & \text{if } p^{\mathbf{r} \star} < p^{\mathbf{r}} \le p^{\mathbf{f}}. \end{cases}$$

Given the linearity of the outer maximization, the optimization is simply solved by comparing the linear coefficient with zero. If  $p^r < p^{r\star}$ , it is more profitable to sell a firm power contract, and the resulting expected profit is obtained by selling the optimal firm power contract  $s^{\star}(0)$ , which coincides with  $p^{r\star}\mu$  as  $p^{r\star}$  is defined as the marginal opportunity cost of selling risky power (i.e., marginal profit of selling firm power). If  $p^r > p^{r\star}$ , it is more profitable to sell a risky power contract, and the resulting profit is  $p^r\mu$ . When  $p^r = p^{r\star}$ , it is indifferent to sell a firm or risky power contract. Consequently any  $\alpha \in [0, 1]$  together with a corresponding optimal firm power contract  $s^{\star}(\alpha)$  are optimal.

## B. Gaussian Forecast Error Case and Price of Unitized Risk

Empirical studies suggest that the forecast error for wind power generation follows a (truncated) Gaussian distribution [5]. Thus when a forecast of the wind power generation is available, W can be represented as a Gaussian random variable centered around its forecast value with variance being the variance of the forecast error. Consequently it is of practical interest to obtain closed-form results if Wis modeled as Gaussian. It is also informative to do so: due to the nice properties of the Gaussian distribution, the dependence of the fair price, optimal contracts, and optimal profit on the mean and variance are more transparent and insightful. The following lemma collects results for the single wind farm contract offering problem in the case for which  $W \sim N(\mu, \sigma^2)$ :

Lemma 3.5: Suppose  $W \sim N(\mu, \sigma^2)$ , and Z is a standard normal random variable. Let  $\phi(\cdot)$  and  $\Phi(\cdot)$  be the probability density function (pdf) and cdf of the standard normal distribution, respectively.

(i) For each fixed  $\alpha$ ,

$$s^{\star}(\alpha) = (1 - \alpha)(\mu + \sigma \Phi^{-1}(\gamma)),$$
$$h(s^{\star}(\alpha), \alpha) = (1 - \alpha)(p^{\mathrm{f}}\mu - q\sigma),$$

where

$$q = -p^{f} \Phi^{-1}(\gamma) + \mathbb{E} \left[ \kappa (\Phi^{-1}(\gamma) - Z)_{+} - \lambda (Z - \Phi^{-1}(\gamma))_{+} \right]$$
  
=  $(\kappa - \lambda) \phi(\Phi^{-1}(\gamma)) > 0.$  (6)

(ii) The fair price for the risky power contract is

$$p^{\mathbf{r}\star} = p^{\mathbf{f}} - q\frac{\sigma}{\mu}.\tag{7}$$

Remark 3.6 (Aggregating i.i.d. wind power sources):

Consider the case in which n independent and identically distributed (i.i.d.) wind power sources, each distributed as  $N(\mu, \sigma^2)$ , are aggregated. The mean and variance of the aggregate wind are  $n\mu$  and  $n\sigma^2$ . Thus the fair price for the aggregate wind power generation is

$$p^{\mathbf{r}\star}_{\ n} = p^{\mathbf{f}} - q \frac{\sigma}{\sqrt{n\mu}} \to p^{\mathbf{f}}$$

as  $n \to \infty$  with a convergence rate in the order of  $1/\sqrt{n}$ .

*Remark 3.7 (Price of unitized risk):* (7) reveals the dependence of the fair price for the risky contract on the forecast and forecast error of W. If  $\sigma = 0$ , i.e., the wind power can be perfectly forecast, the fair price for risky wind power equals the price of firm power; when the unitized risk  $\sigma/\mu$  is large, the fair price for risky power can be much lower than  $p^{\rm f}$ . The quantity q captures the marginal cost of increasing  $\sigma/\mu$ , and thus we term it *the price of unitized risk*. Note that the formula and concept here are related to the price of uncertainty proposed in [16] in the context of stochastic dispatch.

### IV. TWO WIND FARM PROBLEM

In the general case where there are multiple WPPs, wind aggregation provides key benefits to all the WPPs because of the *diversity* among generation from different WPPs. In this section, we study the case of two WPPs in which both of them can trade their future power generation via risky contracts. In particular, each WPP can buy risky power from the other WPP so as to have a better mix of random generation.

#### A. Wind Aggregation via Risky Power Contract

For any set of prices  $p^{f}$ ,  $p_{1}^{r}$  and  $p_{2}^{r}$  (driven by, e.g., supply and demand on the power market), WPP 1 can now offer three types of contracts in the DA market: a) a firm power contract  $s_1$  at price  $p^{f}$ , b) a risky power selling contract  $\alpha_1 W_1$  at price  $p_1^{\rm r}$ , and c) a risky power buying contract  $\beta_2 W_2$ at price  $p_2^r$ . Similarly, WPP 2 can offer a firm power contract  $s_2$  at price  $p^{\rm f}$ , a risky power selling contract  $\alpha_2 W_2$  at price  $p_2^{\rm r}$ , and a risky power buying contract  $\beta_1 W_1$  at price  $p_1^{\rm r}$ . Clearly,  $\beta_1 \leq \alpha_1$  and  $\beta_2 \leq \alpha_2$  must be satisfied, because WPP 1 (WPP 2) cannot buy an amount of risky power from WPP 2 (WPP 1) more than what WPP 2 (WPP 1) sells. Specifically, of the  $\alpha_1 W_1$  risky power sold by WPP 1 (and similarly by WPP 2),  $\beta_1 W_1$  is sold to WPP 2, and the remaining  $(\alpha_1 - \beta_1)W_1$  is sold to other buyers on the market. The DA market model of the two wind farm problem is depicted in Figure 1. In particular, both WPPs first offer their risky power selling contracts  $\alpha_1$  and  $\alpha_2$ . Then each decides on its risky buying and firm contracts  $\{\beta_2 \leq \alpha_2, s_1\}$  and  $\{\beta_1(<\alpha_1), s_2\}$ , respectively, given the other WPP's risky selling contract offering. The RT market operates in the same way as described in the single wind farm case.



Fig. 1: DA market model of the two wind farm problem.

With optimized risky power contracts, each WPP can aggregate random wind from both wind farms to form a more favorable total wind output, and thus earn higher profit overall. Here, we assume that both WPPs have each other's wind forecast information so as to make an informed optimization of wind aggregation via risky power contracts. Such forecast information can be learned from publicly available data, or provided by third party forecast services. Further discussions on relaxing this assumption is provided in Section VII.

#### B. Contract Offering Game

Given a purchase limit  $\alpha_2$  of WPP 2's risky power determined by WPP 2's risky power selling contract, WPP 1 solves the following problem to find its optimal DA firm/risky contract:

$$\max \quad \Pi_1(\alpha_1, \beta_2, s_1) \tag{8a}$$

s.t. 
$$0 \le \alpha_1 \le 1, 0 \le \beta_2 \le \alpha_2$$
 (8b)

where the expected profit of WPP 1 is

$$\Pi_1(\alpha_1,\beta_2,s_1) \tag{9}$$

$$= p_1^{\mathrm{r}} \alpha_1 \mu_1 - p_2^{\mathrm{r}} \beta_2 \mu_2 + p^{\mathrm{f}} s_1 + \mathbb{E} \Big[ -\kappa (s_1 - \widetilde{W}_1(\alpha_1, \beta_2))_+ + \lambda (\widetilde{W}_1(\alpha_1, \beta_2) - s_1)_+ \Big],$$

and

$$\widetilde{W}_1(\alpha_1, \beta_2) = (1 - \alpha_1)W_1 + \beta_2 W_2$$
 (10)

is the total random wind that is available in the RT market for WPP 1 to deliver its firm DA commitment  $s_1$ .

Similarly, given a purchase limit  $\alpha_1$  of WPP 1's risky power, WPP 2 solves the following optimal contract problem:

$$\max \quad \Pi_2(\alpha_2, \beta_1, s_2) \tag{11a}$$

s.t. 
$$0 \le \alpha_2 \le 1, 0 \le \beta_1 \le \alpha_1$$
 (11b)

where  $\Pi_2(\alpha_2, \beta_1, s_2)$  is defined similarly to (9).

Note that the *optimal solutions* of (8a) and (11a) are functions of  $\alpha_2$  and  $\alpha_1$ , respectively, due to the constraints (8b) and (11b). Accordingly, we denote the two sets of optimal risky contracts as  $\alpha_1^*(\alpha_2), \beta_2^*(\alpha_2)$  and  $\alpha_2^*(\alpha_1), \beta_1^*(\alpha_1)$ .

Considering the two WPPs as expected profit maximizers forming a two-player game, WPP 1's and WPP 2's strategies are  $\alpha_1$  and  $\alpha_2$  respectively. The solutions to (8a) and (11a) hence give the *best responses* of WPP 1 and WPP 2 to each other's strategies. We are thus interested in the following questions: a) Does the game have a pure strategy Nash equilibrium, i.e.,  $\{\alpha_1^e, \alpha_2^e\}$  s.t.  $\alpha_1^e = \alpha_1^*(\alpha_2^e)$  and  $\alpha_2^e = \alpha_2^*(\alpha_1^e)$ ? b) Is the equilibrium unique? c) How can the equilibrium solutions be computed and what are their structural properties? We will answer these questions below.

## V. OPTIMAL CONTRACT OFFERING FOR EACH WIND FARM

In this section, we first develop general expressions for the best response of each WPP that maximizes its expected profit given the other WPP's selling strategy. Next, assuming Gaussian distributions of wind output, we obtain closed form solutions for the best responses which reveal clear intuition into each WPP's optimal strategy: Each WPP should either sell all its power via a risky power contract, or optimally and maximally aggregate power bought from the other WPP, depending on which option provides higher expected profit.

Without loss of generality (WLOG), we discuss the best response of WPP 1 given WPP 2's strategy. Similar results for WPP 2's best response can be derived by symmetry.

#### A. General Forms for Best Response of Each WPP

Given WPP 2's strategy  $\alpha_2$ , the optimal contract offering problem of WPP 1 (8a) can be further written as

$$\max_{\substack{\alpha_1 \in [0,1]\\\beta_2 \in [0,\alpha_2]}} p_1^{\mathrm{r}} \alpha_1 \mu_1 - p_2^{\mathrm{r}} \beta_2 \mu_2 + \max_{s_1} \left\{ h(s_1, \alpha_1, \beta_2) \right\} \quad (12)$$

where

$$h(s_1, \alpha_1, \beta_2) = p^{\mathsf{f}} s_1 + \\ \mathbb{E} \Big[ -\kappa (s_1 - \widetilde{W}_1(\alpha_1, \beta_2))_+ + \lambda (\widetilde{W}_1(\alpha_1, \beta_2) - s_1)_+ \Big],$$

with  $\overline{W}_1(\alpha_1, \beta_2)$  given by (10). From Lemma 3.1, the solution to the inner maximization is given by

$$s^{\star}(\alpha_1, \beta_2) = F_{\widetilde{W}_1(\alpha_1, \beta_2)}^{-1}(\gamma) \text{ with } \gamma = \frac{p^{1} - \lambda}{\kappa - \lambda}, \qquad (13)$$

where the inverse cdf is with respect to (w.r.t.) the aggregate wind  $\widetilde{W}_1(\alpha_1, \beta_2)$ . This cdf can be further shown to be

$$F_{\widetilde{W}_1(\alpha_1,\beta_2)}(x) = \mathbb{E}_{W_1}\left[F_{W_2}\left(\frac{x-(1-\alpha)W_1}{\beta_2}\right)\right].$$
 (14)

In general, for arbitrary joint distributions of  $W_1$  and  $W_2$ , the inner maximum  $h(s^*(\alpha_1, \beta_2), \alpha_1, \beta_2)$  cannot be expressed as a simple function of  $\alpha_1$  and  $\beta_2$ . Next, we consider *jointly Gaussian distributions* of  $W_1$  and  $W_2$ , and obtain further insights into the optimal contracts for the two wind farms. Specifically, we denote by  $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$  and  $\rho$  the means, variances and correlation coefficient of wind from WPP 1 and WPP 2.

## B. Convex Optimization for Optimal Risky Contracts

With  $W_1, W_2 \sim N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ , the optimal risky contract problem for WPP 1 (12) simplifies to the following. *Theorem 5.1 (Optimal Risky Contract for Each WPP):* 

Given that the DA risky power selling contract  $\alpha_2$  by WPP 2, the optimal DA risky contract for WPP 1 is a solution to

$$\max_{\alpha_1 \in [0,1], \beta_2 \in [0,\alpha_2]} \Pi(\alpha_1, \beta_2) \tag{15}$$

where

$$\Pi(\alpha_1,\beta_2) = -A\alpha_1 + B\beta_2 - q\sigma(\alpha_1,\beta_2) + p^{\rm f}\mu_1, \quad (16)$$
 with

$$A = p^{\rm f} \mu_1 - p_1^{\rm r} \mu_1, \ B = p^{\rm f} \mu_2 - p_2^{\rm r} \mu_2, \tag{17}$$

$$\sigma^{2}(\alpha_{1},\beta_{2}) = (1-\alpha_{1})^{2}\sigma_{1}^{2} + 2(1-\alpha_{1})\beta_{2}\rho\sigma_{1}\sigma_{2} + \beta_{2}^{2}\sigma_{2}^{2},$$
(18)

and q given by (6).

It can be verified that  $\sigma(\alpha_1, \beta_2)$  is a jointly convex function of  $\alpha_1$  and  $\beta_2$ . Since  $q \ge 0$  (cf. Lemma 3.5), (15) is a *convex optimization* with linear (box) constraints. Furthermore, since  $p^{\rm f} \ge p_1^{\rm r}$  and  $p^{\rm f} \ge p_2^{\rm r}$ , (17) implies that

$$A \ge 0, B \ge 0. \tag{19}$$

## C. Closed Form Solutions for Optimal Risky Contracts

We now show that (15) has closed form solutions that provide insight into the optimal risky contracts. As in Section IV-B, we denote the optimal solutions of (15) as functions of  $\alpha_2$  by  $\{\alpha_1^*(\alpha_2), \beta_2^*(\alpha_2)\}$ .

We begin with the following lemma on the optimal risky *buying* contract of WPP 1.

Lemma 5.2:  $\exists \beta_2^o \in [0, \min\{\frac{\sigma_1}{\sigma_2}, 1\}]$ , such that

$$\beta_2^*(\alpha_2) = \min\{\alpha_2, \beta_2^o\}.$$

From Lemma 5.2, WPP 1 computes a target amount of risky power bought from WPP 2, namely  $\beta_2^o$ , and buys all WPP 2 offers *up to this target amount*.

Accordingly, when  $\alpha_2 \leq \beta_2^o, \, \alpha_1^*(\alpha_2)$  is a solution to

$$\max_{\alpha_1 \in [0,1]} \{ -A\alpha_1 + B\alpha_2 - q\sigma(\alpha_1, \alpha_2) + p^{\rm f}\mu_1 \}.$$
 (21)

Before we state a closed form solution of (21), as well as specifying the target buying amount  $\beta_2^o$ , we first define the following quantities:

$$\eta_1 = q\sigma_1 - A = \mu_1 (p_1^{\rm r} - p_1^{\rm r\star}), \tag{22}$$

$$\eta_2 = q\sigma_2 - B = \mu_2 (p_2^{\rm r} - p_2^{\rm r\star}). \tag{23}$$

In other words,  $\eta_i$  (i = 1, 2) is the additional profit WPP *i* earns by selling *all* its wind energy via a *risky* contract at price  $p_i^{\rm r}$ , compared to selling at the fair price  $p_i^{\rm r\star}$ . Immediately we have the following remark.

*Remark 5.3:* When  $\eta_i \leq 0$ , i.e.,  $p_1^r \leq p_1^{r\star}$ , there is no benefit at all for WPP *i* to sell any portion of its risky power.

This is because, by keeping this portion of risky power, WPP i can always sell an optimal firm power contract based on it, and achieves no less expected profit.

We now have the following theorem on the optimal risky *selling* contract of WPP 1.

Theorem 5.4:  $\alpha_1^*(\alpha_2)$  can be determined as follows:

• When  $\alpha_2 \leq \beta_2^o$ ,  $\alpha_1^*(\alpha_2) = \hat{\alpha}_1(\alpha_2)$  (24)

where

$$\hat{\alpha}_1(\alpha_2) = \begin{cases} 0 & \text{If } \eta_1 \le 0, \\ \min\{(1 - C_1 \alpha_2)_+, 1\}, & \text{If } \eta_1 > 0, \end{cases}$$
(25)

and 
$$C_1 = \frac{\sigma_2}{\sigma_1} \left( A \sqrt{\frac{1-\rho^2}{\eta_1(A+q\sigma_1)}} - \rho \right).$$
 (26)

• When  $\alpha_2 > \beta_2^o$ ,  $\alpha_1^*(\alpha_2) = \alpha_1^*(\beta_2^o)$ .

From (25), when  $\alpha_2 \leq \beta_2^o$  and  $\eta_1 \geq 0$ , WPP 1 computes a target risky power selling contract  $1-C_1\alpha_2$ , and then *applies to it an upper threshold* 1 *and a lower threshold* 0. Theorem 5.4 immediately implies the following.

Corollary 5.5:  $\alpha_1^*(\alpha_2)$  is a non-increasing function of  $\alpha_2$ . The following observation clarifies the intuition behind Corollary 5.5. When  $\eta_1 > 0$ , i.e.,  $p_1^r > p_1^{r*}$ , the only reason for WPP 1 to reserve some portion of its risky power and not sell it via a risky power contract, is to combine its own risky power with that bought from WPP 2 (i.e.,  $\beta_2 W_2$ ) for better statistical behavior of the aggregate wind (e.g., less unitized risk). Thus, as  $\alpha_2$  ( $< \beta_2^o$ ) increases,  $\beta_2^*$  increases (cf. Lemma 5.2), and WPP 1's optimal aggregation strategy is to reserve a greater amount of its own risky power (i.e.,  $(1-\alpha_1)W_1$ ) to mix with  $\beta_2^*W_2$ . We now clarify the meaning of  $C_1$  (cf. (26)).

Remark 5.6 (WPP 1's optimal combining ratio given  $W_2$ ): From (25), given that WPP 1 already purchased an amount  $\alpha_2 W_2$ , it computes an *optimal* amount of its own power to aggregate with  $\alpha_2 W_2$  that maximizes its expected profit. This optimal amount is exactly given by  $(C_1 \alpha_2) W_1$ . Thus, for WPP 1, the optimal combining ratio between its own power and a given amount of WPP 2's power is  $C_1$ .

Finally, we have the following theorem on WPP 1's target risky power buying contract  $\beta_2^o$  used in Lemma 5.2 and Theorem 5.4.

*Theorem 5.7:*  $\beta_2^o$  can be determined as follows:

$$\beta_2^o = \begin{cases} 0, & \text{if } p_1^r \mu_1 \ge \Pi_1(\hat{\alpha}_1(\hat{\beta}_2), \hat{\beta}_2), \\ \hat{\beta}_2 \ (\ge \frac{1}{C_1}), & \text{if } p_1^r \mu_1 < \Pi_1(\hat{\alpha}_1(\hat{\beta}_2), \hat{\beta}_2), \end{cases}$$
(27)

where

(20)

$$\hat{\beta}_{2} = \begin{cases} 0 & \text{If } \eta_{2} \leq 0, \\ \min\{(C_{2})_{+}, 1\}, & \text{If } \eta_{2} > 0, \end{cases}$$
  
and  $C_{2} = \frac{\sigma_{1}}{\sigma_{2}} \left( B \sqrt{\frac{1 - \rho^{2}}{\eta_{2}(B + q\sigma_{1})}} - \rho \right), \qquad (28)$ 

with  $\hat{\alpha}_1$  given by (25),  $C_1$  given by (26), and  $\Pi_1(\alpha_1, \beta_2)$  defined in (16).

We now clarify the meaning of  $C_2$ , which is similar to  $C_1$  but with key differences.



Fig. 2: Type III and Type IV best response of WPP 1:  $\alpha_1^*(\alpha_2)$ .

$\alpha_1^* \backslash \alpha_2^*$	Ι	II	III	IV
Ι	(0,0)	(0, 1)	(0, 1)	(0,1)
II	(1,0)	(1, 1)	$(1, 1 - C_2)$	(1,0)
III	(1,0)	$(1-C_1,1)$	Case III-III	(1,0)
IV	(1,0)	(0, 1)	(0, 1)	Case IV-IV

TABLE I: Equilibria in 16 cases of best response pairs.

Remark 5.8 (WPP 1's optimal combining ratio given  $W_1$ ): Given that WPP 1 already reserved its own power  $W_1$  (i.e., it is not sold as risky power), it computes an *optimal* amount of WPP 2's power to aggregate with  $W_1$  that maximizes its expected profit. This optimal amount is exactly given by  $C_2W_1$ . Thus, for WPP 1, the optimal combining ratio between WPP 2's power and a given amount of its own power is  $C_2$ .

Lemma 5.2, Theorem 5.4 and Theorem 5.7 provide a complete and closed form characterization of the optimal risky power buying and selling contracts for WPP 1. From them, we see that the optimal risky contract offering of WPP 1 can be captured by the following simple strategy:

Remark 5.9 (Sell all vs. maximally aggregate): The optimal risky contract buying and selling strategy for WPP 1 is to compare the expected profit and choose from the following two options. a) Sell all of its power via a risky power contract. b) Compute an optimal combining ratio  $C_2$ ; according to this ratio, buy WPP 2's risky power as much as possible for aggregation until either all its own power is used up ( $\alpha_1 = 0$ ) or all that WPP 2 offers is bought up ( $\beta_2 = \alpha_2$ ).

## VI. EQUILIBRIUM CONTRACT FOR TWO WIND FARMS

#### A. Types of Best Responses and Equilibrium Contract

From Theorems 5.4 and 5.7, the best response functions  $\alpha_1^*(\alpha_2)$  (and similarly  $\alpha_2^*(\alpha_1)$ ) must be one of the following four types:

Definition 6.1 (Four types of best responses):  $\forall \alpha_2 \in [0, 1],$ 

- Type I best response:  $\alpha_1^*(\alpha_2) = 0$ .
- Type II best response:  $\alpha_1^*(\alpha_2) = 1$ .
- Type III best response:  $\alpha_1^*(\alpha_2) = 1 C_1 \alpha_2, C_1 \leq 1$
- Type IV best response:  $\alpha_1^*(\alpha_2) = (1 C_1 \alpha_2)_+, C_1 > 1.$

Type III and IV are illustrated in Figure 2. Furthermore, for a given set of parameters of the WPPs and the prices, the type of the best response is fully determined by Theorems 5.4 and 5.7.

As a result, for finding equilibrium contracts  $(\alpha_1^e, \alpha_2^e)$  for WPP 1 and WPP 2, there are in total 16 cases to consider, which are summarized in the following theorem.

Theorem 6.2: The equilibrium contracts  $(\alpha_1^e, \alpha_2^e)$  that satisfy  $\alpha_1^e = \alpha_1^*(\alpha_2^e)$  and  $\alpha_2^e = \alpha_2^*(\alpha_1^e)$  are specified as in Table I, where Case III-III and Case IV-IV are as follows:

- Case III-III: the case of  $\{C_1 < 1, C_2 \le 1\}$  or  $\{C_2 < 1, C_1 \le 1\}$  does not exist. When  $C_1 = C_2 = 1$ ,  $\forall \alpha \in [0, 1], (\alpha, 1 \alpha)$  is an equilibrium.
- Case IV-IV: there are three equilibria,

$$\left\{(0,1), (1,0), \left(\frac{C_1-1}{C_1C_2-1}, \frac{C_2-1}{C_1C_2-1}\right)\right\}.$$
 (29)

As an example, we evaluate as follows the equilibrium contracts for the special cases of fully correlated wind.

*Example 6.3 (Fully positively correlated wind):* If  $\rho = 1$ , the equilibrium risky contracts are

$$(\alpha_1^e, \beta_2^e, \alpha_2^e, \beta_1^e) = \begin{cases} (1, 0, 1, 0) & \text{if } \eta_1 > 0, \eta_2 > 0, \\ (1, 0, 0, 0) & \text{if } \eta_1 > 0, \eta_2 < 0, \\ (0, 0, 1, 0) & \text{if } \eta_1 < 0, \eta_2 > 0, \\ (0, 0, 0, 0) & \text{if } \eta_1 < 0, \eta_2 < 0. \end{cases}$$

Since the wind power processes at both wind farms are fully correlated, no statistical diversity can be leveraged via wind aggregation. As the optimization problem for each WPP to decide its optimal risky contracts (cf. (15)) reduces to linear programming, the best responses (and consequently equilibria) are straightforwardly implied by the single WPP solutions as derived in Section III.

*Example 6.4 (Fully negatively correlated wind):* If  $\rho = -1$ , WLOG assume  $\sigma_1 \leq \sigma_2$ , the equilibrium risky contracts are

$$(\alpha_1^e, \beta_2^e, \alpha_2^e, \beta_1^e) = \begin{cases} (0, \sigma_1/\sigma_2, 1, 0) & \text{if } \eta_1 > 0, \eta_2 > 0, \\ (1, 0, 0, 1) & \text{if } \eta_1 > 0, \eta_2 < 0, \\ (0, \sigma_1/\sigma_2, 1, 0) & \text{if } \eta_1 < 0, \eta_2 > 0, \\ (0, 0, 0, 0) & \text{if } \eta_1 < 0, \eta_2 < 0. \end{cases}$$

Note that, when  $\sigma_1 < \sigma_2$ , a nontrivial acquisition by WPP 1 of WPP 2's risky power  $0 < \beta_2^e < \alpha_2^e$  is in an equilibrium under the price conditions  $\eta_1 > 0$ ,  $\eta_2 > 0$ .

Next, we focus on the particular case of  $C_1 = C_2 = 1$ , as it leads to an interesting generalization of the "fair price" concept from the single wind farm case (cf. (7)) to the two wind farm case.

# B. The Case of $C_1 = C_2 = 1$ and Generalized Fair Price

From Theorem 6.2, when  $C_1 = C_2 = 1$ ,  $\forall \alpha \in [0,1]$ ,  $(\alpha_1 = \alpha, \alpha_2 = 1 - \alpha)$  is an equilibrium pair of strategies of WPP 1 and WPP 2. From the definitions of  $C_1$  and  $C_2$  in (26) and (28),  $C_1 = C_2 = 1$  is achieved when a)  $\rho \geq \max\{-\frac{\sigma_1}{\sigma_2}, -\frac{\sigma_2}{\sigma_1}\}$  and b) the prices of risky power of WPP 1 and WPP 2 satisfy

$$p_1^{\mathbf{r}} = \tilde{p}_1^{\mathbf{r}\star} \triangleq p^{\mathbf{f}} - q \frac{\sigma_1}{\mu_1} \cdot \frac{\sigma_1 + \rho \sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho \sigma_1 \sigma_2}}, \qquad (30)$$

$$p_2^{\mathbf{r}} = \tilde{p}_2^{\mathbf{r}\star} \triangleq p^{\mathbf{f}} - q \frac{\sigma_2}{\mu_2} \cdot \frac{\sigma_2 + \rho \sigma_1}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}}.$$
 (31)

We then have the following lemma on the *indifference* among the equilibrium strategies and their *equivalence to full wind aggregation (coalition)*:

*Lemma 6.5:* When  $\rho \ge \max\{-\frac{\sigma_1}{\sigma_2}, -\frac{\sigma_2}{\sigma_1}\}$ , if (30) and (31) hold, then  $\forall \alpha \in [0, 1]$ ,

With equilibrium strategies (α<sub>1</sub> = α, α<sub>2</sub> = 1-α), WPP 1 and WPP 2 achieve invariant amounts of expected profits p<sup>r</sup><sub>1</sub>μ<sub>1</sub> and p<sup>r</sup><sub>2</sub>μ<sub>2</sub> respectively, regardless of α.
The sum of the two WPPs' expected profits satisfies

$$p_1^{\rm r}\mu_1 + p_2^{\rm r}\mu_2 = p_{\{1,2\}}^{\rm r\star}\mu_{\{1,2\}},\tag{32}$$

where  $p_{\{1,2\}}^{r\star} = p^f - q_{\mu_{\{1,2\}}}^{\sigma_{\{1,2\}}}$ , with  $\mu_{\{1,2\}} = \mu_1 + \mu_2$  and  $\sigma_{\{1,2\}}^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$  being the mean and the variance of the aggregation of *all the wind from both WPPs*.

Note that the right hand side of (32) equals the expected profit of first combining the two WPPs into a single large WPP (a coalition), and then selling an optimal firm power contract based on this fully aggregated wind.

In light of Lemma 6.5, we can generalize the definitions of *fair prices* for WPP 1 and WPP 2 to be (30) and (31), denoted by the pair  $(\tilde{p}_1^{r*}, \tilde{p}_2^{r*})$ . Note the interesting analogy between (30), (31) and (7): Because of the presence of the two WPPs and hence the possibility of wind aggregation, the fair prices for both WPPs become higher. This change is captured by the last terms in (30) and (31) as the "correction terms", and indicates the *value of wind aggregation*.

Remark 6.6 (Sharing profit within a coalition): From Lemma 6.5, we see that, with generalized fair prices  $\tilde{p}_1^{r\star}$ and  $\tilde{p}_2^{r\star}$ , two WPPs collectively earn a total profit equal to which can be earned by directly forming a coalition. These generalized fair prices thus indicate a reasonable mechanism for sharing profit for two WPPs, if a coalition of the two is indeed formed. Specifically, WPP 1 and WPP 2 earn  $\tilde{p}_1^{r\star}\mu_1$ and  $\tilde{p}_2^{r\star}\mu_2$  respectively.

## C. Numerical Examples

We next provide numerical examples that demonstrate a clear picture of the equilibrium risky power contracts and their efficiency (i.e., Pareto optimality) as prices change. We consider two WPPs with  $\mu_1 = \mu_2 = \mu$ ,  $\sigma_1 = \sigma_2 = \sigma = 0.2\mu$  (i.e., a unitized risk of 0.2), and  $\rho = -0.5$  (i.e., somewhat negatively correlated wind). We consider the real time penalty (price)  $\kappa$  to be 1.6 times the DA firm power price  $p^{\rm f}$ , and  $\lambda = 0$ . These imply a price of unitized risk (cf. (6)) q = 0.6. The fair price for each single WPP (cf. (7)) is thus  $p^{\rm r}{}_1^* = p^{\rm r}{}_2^* = p^{\rm r}{}^* = p^{\rm f} - 0.2q = 0.88$ .

We now evaluate the equilibrium contracts for WPP 1 and WPP 2 for price  $p_1^r = p_2^r = p^r$  ranging from  $p^{r*}$  to  $p^f$ . Specifically, the total profits (normalized by the profit gained if the wind forecast error variance is zero) of the two WPPs are plotted in Figure 3 for the following three cases:

- Each WPP sells all its power via risky contracts.
- $\alpha_1 = \alpha_2 = \frac{C-1}{C^2-1}$  (cf. (29)), where  $C = C_1 = C_2$  (cf. (26) and (28)).
- Two WPPs form a coalition, and optimally sell a firm power contract.



Fig. 3: Total profits achieved at equilibria for different prices.

We observe that the three curves coincide at  $p^{\rm r} = \tilde{p}^{\rm r\star} = p^{\rm f} - 0.2q \sqrt{\frac{1+\rho}{2}} = 0.94$  (cf. (30)).

- When p<sup>r</sup> > p̃<sup>r\*</sup>, C < 1, and selling all power via a risky power contract is optimal for both WPPs as indicated in Theorem 5.7. The increasing line to the right of p̃<sup>r\*</sup> corresponds to these equilibria.
- When  $p^r < \tilde{p}^{r\star}$ , C > 1, and there are three equilibria for each particular  $p^r$  as indicated in Theorem 6.2: in addition to the equilibrium  $\alpha_1 = \alpha_2 = \frac{C-1}{C^2-1}$ , the equilibria of  $(\alpha_1, \alpha_2)$  equal to (0, 1) and (1, 0) coincide with the coalition curve.

Note that the entire equilibria curve is above  $\frac{p^{r\star}}{p^{f}} = 0.88$ , meaning that both WPPs prefer participating in the risky power market to just selling firm power on their own. As indicated in Theorem 6.5, when  $p^{r} = \tilde{p}^{r\star}$ , C = 1, and all the equilibria reach the coalition curve (i.e., Pareto optimality). Moreover, the two WPPs earn the same amount of profit, which is intuitively fair because the two WPPs have the same statistics. When  $p^{\rm r} < \tilde{p}^{\rm r\star}$ , however, the equilibrium  $\left(\frac{C-1}{C^2-1}, \frac{C-1}{C^2-1}\right)$  does *not* reach the coalition curve. While the equilibria (1,0) and (0,1) do reach the coalition curve, the profits earned by the two WPPs are unequal, and hence *not* fair. Lastly, the case of  $p^{r} < \tilde{p}^{r\star}$  leads to both WPPs selling all of their power, because the price offered to each of them on the market is so high that the two WPPs would not earn a higher profit even if they formed a coalition and sold firm power optimally.

## VII. CONCLUSIONS AND FUTURE WORK

We have proposed introducing risky power contracts in addition to firm power contracts to enable flexible and efficient wind power aggregation. Starting with the case of a single WPP, we have derived analytical solutions for the optimal risky and firm power contracts. Based on the obtained dependence of optimal contracts on prices, the concept of the fair price of a single WPP and that of the price of unitized risk have been introduced. We have then studied the more general problem in which there are two WPPs both trading their risky power in addition to firm power. We have derived analytical solutions for the optimal risky and firm power contracts for each WPP given the other WPP's contract offering: The simple strategy of either sell all or optimally and maximally aggregate wind is optimal. Based on the derived best responses for each WPP, we have characterized analytically the equilibrium contracts for all cases of prices. The equilibrium results have led us to a generalization of the fair prices to the two WPP setting, which characterizes the value of wind aggregation. With the generalized fair prices, all equilibria achieve the same total profit as forming a coalition of the two wind farms. The generalized fair prices further indicate a reasonable mechanism for profit sharing for the case in which the two WPPs form a coalition.

The focus of this paper has been placed upon how optimal offerings and equilibria depend on exogenous price signals, and upon deriving concept and expressions for critical prices from the perspective of WPPs. The following questions that are important for the generalization and implementation of risky power contracts are left for future research.

(a) What is a 'good' pricing scheme for risky power contracts given the statistics of the wind power generation?

The question is partially addressed in this work, i.e., only from the WPPs' perspective. Based upon our definition of the fair price, for a single WPP, it is only incentivized to sell in the risky power market if  $p^r \ge p^{r\star}$ . A similar concept is developed for the two WPP case, where we show how generalized fair prices depend on the joint statistics of both wind processes. However, as fair prices defined here do not result from a market mechanism, it is not clear how setting  $p^r = p^{r\star}$  will enforce any kind of social efficiency. Existing research has analyzed the problem of wind power pricing in traditional market setting without the option of risky power contract [17], [18]. For our formalism, a more detailed model for buyers of risky power in the market would be necessary to study pricing questions of this nature.

(b) How can risky power markets be implemented when there are  $N \ge 2$  WPPs?

To simplify the presentation, this paper has focused on the case of two WPPs. Generalization to multiple WPPs is possible, but will require a specification of market architecture when multiple buyers are competing for a specific type of risky power supply. Interested readers are referred to [19] for auction schemes that are designed to enable wind power aggregators to efficiently aggregate wind power generation from different sources. While it is not hard to give a naive scheme that works for the N-WPP case, developing schemes that enjoy nice economic properties such as incentive compatibility will require a closer investigation.

(c) How can one ensure the knowledge of wind forecast and its statistics is truthful if the information is reported by WPPs themselves?

This paper makes the assumption that the statistics of wind power generation are public, as in [13]. This assumption may not hold if the wind power statistics are available only through WPPs' own forecasts. In such cases, correctly designed market mechanisms must ensure incentives for WPPs to truthfully report their forecasts and statistics of forecast errors (cf. [19] for an example). Nevertheless, the results of our paper suggest a clear benefit of aggregation, which may encourage third party companies to enter the business of providing wind forecast services to enable risky power trading discussed in this paper (if the market allows these third party companies to share part of the benefit of aggregation). Such possibilities can provide a ground for foreseeable opportunities and flexibilities brought to the power markets by risky power contracts.

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