New Formulation and Optimization Methods for Water Sensor Placement

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Abstract

Optimal sensor placement for detecting contamination events in water distribution systems is a well explored problem in water distribution systems security. We study herein the problem of sensor placement in water networks to minimize the consumption of contaminated water prior to contamination detection. For any sensor placement, the average consumption of contaminated water prior to event detection amongst all simulated events is employed as the sensing performance metric. A branch and bound sensor placement algorithm is proposed based on greedy heuristics and convex relaxation. Compared to the state of the art results of the battle of the water sensor networks (BWSN) study, the proposed methodology demonstrated a significant performance enhancement, in particular by applying greedy heuristics to repeated sampling of random subsets of events.

I. Introduction

A water distribution system (WDS) is a basic infrastructure in modern society. Contamination events generated in the network can risk many lives within minutes. This underlines the importance of gaining knowledge regarding detection of such events within a WDS. Contamination of water networks can be caused by intentional and unintentional events. The necessity to connect to

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all consumers causes WDSs to be spatially dispersed so that supervision and control over central pipelines is not economically viable. In addition, many unintentional contamination events may not be identified by only grab sampling monitoring. These demonstrate the need to place sensors, which measure quality parameters in water distribution systems, to allow detection of such cases and prevent further consumption of contaminated water.

Currently, there are limited predefined actions in case of a contamination event in water networks (Hart and Murray, 2010). The assumption is that in case of an event a "Do Not Use" order will likely be declared first which can evolve to a shutdown of the entire system in extreme circumstances. The evolution of sensor deployment and assessment in the network may lead to the option of isolating the contaminant and shutting off only the polluted region. This would allow solving for the optimal performance of a network with specific links and consumers taken out of the network operation.

Optimal sensor placement integrates two conflicting intentions: insuring the health and safety of the population, on the one hand, and satisfying the budget and other constraints which define the number of sensors assigned to the project, on the other (Berry et al., 2005; Ostfeld et al., 2008; Berry et. al., 2008b; Hart 2008). As a result, sensor placement optimization can be used to lower the degree of impact on consumers, keeping it to a minimum.

We study placing sensors to minimize the consumption of contaminated water prior to detection of the contamination. Given any sensor placement, the average consumption of contaminated water prior to event detection among all simulated events is employed as the sensing performance metric. We then transform the sensor placement problem into a mixed integer-convex programming (MICP) problem. A greedy heuristic and a branch and bound algorithm are developed and implemented on two benchmark battle of the water sensor networks (BWSN) example applications (Ostfeld et al., 2008). We discovered that our method is greatly competitive, and improves the state of the art results.

II. Literature Review

Intentional injection of contaminants in WDSs can be attained with relatively simple equipment. The only constraint is to be able to overcome the relatively low pressure in the network. Contamination events in water distribution systems are not particularly common, yet several have been reported (Hickman, 1999; Denininger, 2000; Craun et al., 1991; Clark et al., 1996; HACH, 2011; Whelton et al., 2014; Ha'aretz, 2014).

As an example, one such recent event is the West Virginia's Elk River contamination incident (Whelton et al., 2014). In that occurrence an industrial solvent contaminated the Elk River, causing about 300,000 residents to detach from access to potable water. The chemical spill originated upstream from the principal water distribution system treatment plant intake. A Do Not Use order was called shortly after the event, which lasted for about ten days. Post the immediate consequences, investigations were still ongoing and the communities were still recovering more than eleven months after the incident.

Contamination events can be detected by sensors placed within the network. The state of the art sensor packages include a set of sensors and a device with computational and analytical capabilities. The sensors measure indicative water quality parameters such as pH, residual chlorine, turbidity, and electrical conductivity. The data collected by the sensors is then analyzed to determine if there is an indication of an event alert. This is the so called event detection problem which attracted substantial interest in recent years (e.g., EPA, 2012 through CANARY; Perelman et al., 2012; Arad et al., 2013; Oliker and Ostfeld, 2014a, 2014b).

As sensors cannot be placed at every system node as of budget and other constraints, only a small set of monitoring stations can be deployed. This reality led to the development of numerous algorithms which attempt to find the optimal locations of a limited number of sensors within a WDS.

The first study on sensor placement is attributed to Lee et al. (1991). Lee et al. (1991), Lee and Deininger (1992), and Kumar et al. (1997) solved the sensor placement problem with use of mixed integer linear programming (MILP). The objective function was defined as the maximum demand coverage and steady state flows were assumed in the network. Later on Harmant et. al (1999), Woo et. al (2001), Propato et. Al (2005) and Berry et. al (2005) modified the demand coverage to include water quality and time dependence in the demands. With the introduction of heuristic optimization, Al-Zahrani et al. (2001, 2003) employed genetic algorithms to solve the same problem formulation.

Following Lee et al. (1991), Kessler et al. (1998) added to the coverage problem an all shortest path algorithm and the construction of a pollution matrix. The pollution matrix contained data regarding the nodes which are contaminated by many possible contamination events. The definition of a contaminated node in their study refers to a contaminant concentration which is above a predefined threshold. Ostfeld and Salomons (2004) expanded the study of Kessler et al. (1998)

by introducing a randomized pollution matrix (RPM) which holds a set of randomly generated multiple injection locations and times. The values in the matrix are binary and determine if a node was contaminated at a specific random event. The RPM concept is also used in this study.

Other studies suggested different concepts for sensor placement. Cozzolino et. al (2006) formulated a Lagrangian advection method which estimates the time of each flow path in the network. Chastain (2006) created a database of responses to contamination events at each node in the network. Berry et al. (2008a) formulated a low memory Lagrangian relaxation method which simplified the original problem by assigning a penalty to constraint violation. Berry et al. (2008b) and Hart (2008) presented the Teva Spot toolkit which combines general purpose heuristics with integer programming and bounding algorithms. Several others are based on risk assessment to the consumers (Propato, 2006; Kraus et al. 2006, 2008), data uncertainties (Chastain, 2004; Carr et al., 2004, 2006), imperfect sensors (Wu et al., 2008; Preis and Ostfeld, 2008a; Berry et al., 2006), and multi-objective formulations (Aral et al., 2008; Dorini et al., 2006; Preis and Ostfeld, 2008b).

A benchmark in sensor placement modeling development is the battle of the water sensor networks (BWSN) study held in Cincinnati in 2006 (Ostfeld et al., 2008). The BWSN compared fifteen different approaches of optimal sensor placement in water distribution systems. The approaches considered four objectives: (1) the time of detection; (2) affected population prior to detection; (3) consumption of contaminated water prior to detection; and (4) likelihood of detection. This work is using the networks and outcomes of the BWSN for comparing and evaluating the current developed model and algorithms.

Research on sensor placement is still ongoing: Chang et. al (2012) formulated a rule-based decision support system which consists of two rules: (1) Accessibility; and (2) complexity which is parallel to the optional path calculations from previous articles. Diao and Rauch (2013) presented a controllability analysis of the network which determines the nodes that have an outcome indication over a maximum number of downstream nodes.

III. Problem Formulation

We consider placing sensors in water distribution networks for detecting water contamination. For any contamination event, we are interested in the consumption of the contaminated water prior to detection of the event. The RPM which is constructed as part of the problem, holds a risk

assessment for many contamination events. Each column in the matrix represents a contamination event and each row, a network node. Each value of the matrix represents an assessment of the consumption of the contaminated water assuming that a sensor in the related node is the first to detect the event. For example: if the value in row two column four is 100, then the fourth event will have 100 gallons of the contaminated water consumed before the contaminated water flows into node two and then would immediately be detected by the sensor located there. The matrix results are according to simultaneous simulations with EPANET and EPANET-MSX which calculate the contaminant concentration within the network over time. No assumptions were made as to the risk injection locations to this network and therefore all network nodes were assumed to have an even probability for a contamination event.

We denote the set of all possible sensor locations by $\mathcal{N} = \{1, 2, \ldots, N\}$, and a set of simulated contamination events by $\mathcal{E} = \{E_1, E_2, \ldots, E_K\}$. We define a matrix $G \in \mathbb{R}^{N \times K}$, in which g_{nk} is the consumption of the contaminated water prior to detection of contamination event E_k at node n, if E_k happens.

A. Optimal Sensor Placement

Given a set of M sensors $\mathcal{M} \subseteq \mathcal{N}$, for any event E_k, we consider the detection time of this event to be the *minimum* detection time among these sensors. This is because, E_k will be detected as long as it is detected by any one of the sensors in $\mathcal{M}$. Consequently, the consumption of the contaminated water prior to event detection by the sensors in $\mathcal{M}$ is $\min_{m \in \mathcal{M}} g_{mk}$. For evaluating the performance by the set of sensors $\mathcal{M}$ on detecting all the K events, we employ an average-impact criterion as follows:

$$g^{avg}(\mathcal{M}) = \frac{1}{K} \sum_{E_k \in \mathcal{E}} \min_{m \in \mathcal{M}} g_{mk}. \tag{1}$$

In other words, we consider the average consumption of the contaminated water among all events as the performance metric.

With a limited number of sensors, we investigate which M among the N locations to place sensors in order to optimize contamination detection performance. Based on the average-impact metric (1), we formulate the optimal sensor placement problem as follows:

$$\min_{\mathcal{M} \subseteq \mathcal{N}} \; g^{avg}(\mathcal{M}) \tag{2}$$

$$s.t. \; |\mathcal{M}| \leq M, \tag{3}$$

where $|\cdot|$ denotes the cardinality of a set. Clearly, (2) is a combinatorial optimization problem, for which an exhaustive search will render a exponential computational complexity of $\binom{N}{M}$.

B. Approximation of the Optimization Problem

To overcome the combinatorial complexity of (2), we first employ the following approximation of $\min_{m\in\mathcal{M}} g_{mk}$ in (1), setting up for a mixed binary integer convex programming (MICP) that follows. Specifically,

$$\min_{m\in\mathcal{M}} g_{mk} \approx -\beta \log\left(\sum_{m\in\mathcal{M}} \exp\left(-\frac{g_{mk}}{\beta}\right)\right). \tag{4}$$

where $\beta > 0$ is a tunable parameter. The rationale of this approximation is that the $\log$-$\sum$-$\exp$ function is a differentiable approximation of the non-differentiable $\max$ function (Boyd and Vandenberghe (2004)). In particular, the lower β is, the better approximation we have, and *as β goes to zero, this approximation becomes exact*. In practice, however, β cannot be chosen arbitrarily close to zero for algorithmic reasons (Boyd and Vandenberghe (2004)). In addition, we need a minimum numerical precision of $\exp\left(-\frac{g_{mk}}{\beta}\right)$: if β is too small, $\exp\left(-\frac{g_{mk}}{\beta}\right)$ will go below machine precision and be rounded to exactly zero. A comprehensive discussion of the choice of β is given later in Section V-C.

Next, we introduce a sensor location indicator vector $\boldsymbol{w} \in \mathbb{R}^{N\times 1}$, in which

$$\forall n = 1, 2, \ldots, N,\ w_n = \begin{cases} 1, \text{ if } n \in \mathcal{M}, \text{ i.e., there is a sensor at location } n, \\ 0, \text{ otherwise.} \end{cases} \tag{5}$$

Accordingly, we have

$$\sum_{m\in\mathcal{M}} \exp\left(-\frac{g_{mk}}{\beta}\right) = \sum_{n=1}^{N} w_n \exp\left(-\frac{g_{nk}}{\beta}\right). \tag{6}$$

Substituting (6) and (4) into (1) and (2), and rewriting the constraint on the number of sensors (3) in terms of $\boldsymbol{w}$, we arrive at the following optimization problem,

$$\begin{aligned} \min_{w}\ & -\frac{\beta}{K}\sum_{E_k\in\mathcal{E}} \log\left(\sum_{n=1}^{N} w_n \exp\left(-\frac{g_{nk}}{\beta}\right)\right) \\ s.t.\ & \sum_{n=1}^{N} w_n \le M,\ w_n \in \{0,1\}. \end{aligned} \tag{7}$$

Note that the objective (7) is *convex* in $\boldsymbol{w}$. Because $\boldsymbol{w}$ is a binary vector, solving (7) is a MICP which requires a worst-case exponential computational complexity. In the next section, we will

Algorithm 1: Greedy Sensor Placement

Initialize the set of sensor locations $\mathcal{M}_0 = \emptyset$,

and the number of sensors $m = 0$.

Repeat

$m \leftarrow m + 1$,

$$\mathcal{M}_m = \mathcal{M}_{m-1} \cup \left\{ \underset{n \in \mathcal{N} \setminus \mathcal{M}_{m-1}}{\operatorname{argmin}} \; g^{avg}\left(\mathcal{M}_{m-1} \cup \{n\}\right) \right\} \quad (9)$$

Until $m = M$.

see that it allows a convex relaxation, and eventually a branch and bound algorithm that can effectively solve for its global optimum.

IV. Sensor Placement Algorithm Design

A. A Greedy Heuristic

We first develop a low-complexity greedy sensor placement heuristic as in Algorithm 1 for optimizing the original objective (2). It generates a series of sensor locations $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_N$ for $M = 1, 2, \ldots, N$, respectively, that satisfy the following consistency property:

$$\mathcal{M}_1 \subset \mathcal{M}_2 \subset \ldots \subset \mathcal{M}_N. \quad (8)$$

In other words, given the total number of sensors M, we choose sensor locations *one by one*: At each step, we keep the already chosen locations; from the remaining locations, we choose the one that *minimizes the current step's average consumption of the contaminated water prior to contamination event detection*, and include it in the set of the chosen locations. We note that this greedy heuristic can also be applied to the approximate formulation (7), which will be used as a building block in the branch and bound algorithm development next.

B. A Branch and Bound Algorithm

Toward finding *globally* optimal sensor locations with low computational complexity, we begin with the approximate formulation (7). Since (7) has a convex objective function in w, it has the following *relaxation* as a convex optimization:

$$\min_{w} \quad -\frac{\beta}{K}\sum_{E_k\in\mathcal{E}}\log\left(\sum_{n=1}^{N} w_n \exp\left(-\frac{g_{nk}}{\beta}\right)\right) \tag{10}$$
$$s.t.\ \sum_{n=1}^{N} w_n \le M,\ 0 \le w_n \le 1.$$

Accordingly, the optimal value of (10) serves as a *lower bound*, denoted by L_1, on the global optimum of (7). Meanwhile, the same as in Algorithm 1, a greedy heuristic applied to (7) provides an *upper bound* on its global optimum, denoted by U_1.

For any location j, (7) can be split into two sub-problems by fixing w_j to be either 0 or 1:

$$\min_{w} \quad -\frac{\beta}{K}\sum_{E_k\in\mathcal{E}}\log\left(\sum_{n=1}^{N} w_n \exp\left(-\frac{g_{nk}}{\beta}\right)\right) \tag{11}$$
$$s.t.\ \sum_{n=1}^{N} w_n \le M,\ w_n \in \{0,1\},$$
$$w_j = 0.$$

and

$$\min_{w} \quad -\frac{\beta}{K}\sum_{E_k\in\mathcal{E}}\log\left(\sum_{n=1}^{N} w_n \exp\left(-\frac{g_{nk}}{\beta}\right)\right) \tag{12}$$
$$s.t.\ \sum_{n=1}^{N} w_n \le M,\ w_n \in \{0,1\},$$
$$w_j = 1.$$

Similarly to (10), relaxations of these two sub-problems can be formed by replacing $w_n \in \{0,1\}$ with $0 \le w_n \le 1$, and they provide lower bounds, denoted by $l_2^{(0)}$ and $l_2^{(1)}$, on the global optima of (11) and (12) respectively. Meanwhile, applying the greedy heuristic under the constraint $w_j = 0$ or $w_j = 1$ provides upper bounds, denoted by $u_2^{(0)}$ and $u_2^{(1)}$, on these sub-problems' global optima. Define

$$L_2 \triangleq \max\{l_2^{(0)}, l_2^{(1)}\},\ \text{and}\ U_2 \triangleq \max\{u_2^{(0)}, u_2^{(1)}\}. \tag{13}$$

Then, L_2 and U_2 are new lower and upper bounds on the global optimum of (7) (Balakrishnan et al. (1991)).

More generally, the above splitting procedure with relaxations and greedy heuristics can be applied on the sub-problems themselves to form more children sub-problems with lower and

upper bounds. For example, for any location $s, (s \neq j)$, (11) can be further split into two sub-problems by adding yet another constraint $w_s = 0$ or $w_s = 1$ respectively.

We define the following lower and upper bounding oracles, as well as an oracle that returns the next location to split:

Definition 1: Oracle $LB(\mathcal{C})$ takes a *constraint set* $\mathcal{C}$ as input, where $\mathcal{C}$ specifies a set of locations whose sensor indicator variables are pre-determined to be either 0 or 1. An MICP under the constraints $\mathcal{C}$ is formed, a relaxation is solved, and the optimum of this relaxation is output by $LB(\mathcal{C})$ as a lower bound on the optimum of the constrained MICP.

For example, in (11) and (12), the constraint sets are $\mathcal{C}^{(0)} = \{w_j = 0\}$ and $\mathcal{C}^{(1)} = \{w_j = 1\}$, respectively.

Definition 2: Oracle $UB(\mathcal{C})$ takes a constraint set $\mathcal{C}$ as input. An MICP under the constraints $\mathcal{C}$ is formed, a feasible sensor placement solution is found by a greedy heuristic, and the achieved objective value is output by $UB(\mathcal{C})$ as an upper bound on the optimum of the constrained MICP.

Definition 3: Based on the *order* of the locations chosen by the greedy heuristic, Oracle $next(\mathcal{C})$ outputs *the first* location that is chosen by this heuristic.

When a sub-problem with constraints $\mathcal{C}$ needs to be split further, $next(\mathcal{C})$ is the location we choose to perform the splitting by fixing $w_{next(\mathcal{C})}$ to be either 0 or 1.

We now provide a branch and bound algorithm as in Algorithm 2 where $i_{\max}$ is the maximum number of iterations allowed. As the algorithm progresses, *a binary tree is developed where each node represents a constraint set*. The *leaf nodes* are kept in $\mathcal{S}$. The tree starts with a single node, $\{\emptyset\}$, corresponding to no initial constraint. When a sub-problem corresponding to a leaf node $\mathcal{C}^*$ is split into two new sub-problems, the two new constraint sets $\mathcal{C}^{(0)}$ and $\mathcal{C}^{(1)}$ become the children of the parent constraint set $\mathcal{C}^*$.

In Algorithm 2, (15) is a generalization of (13). It means that the current global lower bound equals the *lowest* lower bound among all the *leaf node* constraint sets. This is true because all the leaf nodes $\mathcal{S}$ represent a *complete partition* of the original parameter space. At the beginning of every iteration, in choosing which leaf node to split (14), we select the one that gives the *lowest* lower bound (i.e., the current *global* lower bound). It is a heuristic based on the reasoning that, by further splitting this critical leaf node, a higher global lower bound may be obtained, (whereas splitting any other node will leave the global lower bound unchanged.) At iteration i, the current lower and upper bounds on the global optimum are available as L_i and U_i. When

Algorithm 2:

Sensor Placement using Branch and Bound

Initial step: $i = 1$,

the initial constraint set: $\mathcal{C}_1 = \emptyset$,

the initial set of leaves of the tree of constraint sets

(initially a single node): $\mathcal{S} = \{\mathcal{C}_1\}$.

Compute $L_1 = LB(\mathcal{C}_1),\ U_1 = UB(\mathcal{C}_1)$.

While $U_i - L_i > \epsilon$ or $i < i_{\max}$, repeat

Choose which leaf node constraint set to split:

$$\mathcal{C}^* = \underset{\mathcal{C}\in\mathcal{S}}{\operatorname{argmin}}\{LB(\mathcal{C})\}. \tag{14}$$

Choose the next location to split, $j = next(\mathcal{C}^*)$,

Form two new constraint sets,

$\mathcal{C}_{i+1}^{(0)} = \mathcal{C}^* \cup \{w_j = 0\},\ \mathcal{C}_{i+1}^{(1)} = \mathcal{C}^* \cup \{w_j = 1\}$.

In the set of leaves $\mathcal{S}$, replace the parent constraint set $\mathcal{C}^*$ with the two children $\mathcal{C}_{i+1}^{(0)}$ and $\mathcal{C}_{i+1}^{(1)}$:

$\mathcal{S} \leftarrow (\mathcal{S}\backslash\{\mathcal{C}^*\}) \cup \{\mathcal{C}_{i+1}^{(0)}\} \cup \{\mathcal{C}_{i+1}^{(1)}\}$.

Compute new lower and upper bounds for the two new constrained MICP:

$LB(\mathcal{C}_{i+1}^{(0)}),\ LB(\mathcal{C}_{i+1}^{(1)}),\ UB(\mathcal{C}_{i+1}^{(0)}),\ UB(\mathcal{C}_{i+1}^{(1)})$,

Update the global lower and upper bounds,

$$L_{i+1} = \min_{\mathcal{C}\in\mathcal{S}}\{LB(\mathcal{C})\},\ U_{i+1} = \min_{\mathcal{C}\in\mathcal{S}}\{UB(\mathcal{C})\}. \tag{15}$$

$i \leftarrow i + 1$.

Choose the best achieved solution so far:

$\hat{\mathcal{C}} = \operatorname{argmin}_{\mathcal{C}\in\mathsf{S}} UB(\mathcal{C})$.

Return the greedy solution under the constraint set $\hat{\mathcal{C}}$.

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these two bounds meet, i.e., $U_i - L_i < \epsilon$, the solution that achieves the current upper bound is guaranteed to be globally optimal.

As the total number of possible constraint sets is 2^N (corresponding to the 2^N sensor location indicator vectors $\boldsymbol{w}$), Algorithm 2 is guaranteed to converge in 2^N iterations (and in practice much less as will be shown later.) To limit the algorithm's run time, a maximum number of

iterations $i_{\max}$ can be enforced as in Algorithm 2.

Finally, we define i_{achieve} to be the number of iterations used to *achieve* the globally optimal solution, and i_{prove} the number of iterations used to *prove* its global optimality. In other words, it takes i_{achieve} iterations for the *upper bound* to reach the global optimum, while it takes i_{prove} iterations for *both* the upper and lower bounds to reach the global optimum.

V. Performance Evaluation

In this section, we evaluate the proposed sensor placement algorithms in two representative water networks that have served as the benchmarks in the Battle of the Water Sensor Networks (BWSN) (Ostfeld et al., 2008), a previously held sensor placement design competition that represents the state of the art.

A. Network and Event Data Description

The BWSN competition provides two benchmark systems: a 129-node network (BWSN1) and a 12527-node network (BWSN2). The topologies of the two networks are depicted in Figure 1. More than 37000 events are simulated for the 129-node network, and more than 25000 events for the 12527-node network. All further details are summarized in Ostfeld et al. (2008). We also attach as supplementary files the two BWSN network example applications we used as .inp EPANET files. The reader can thus follow all the detailed results presented below.

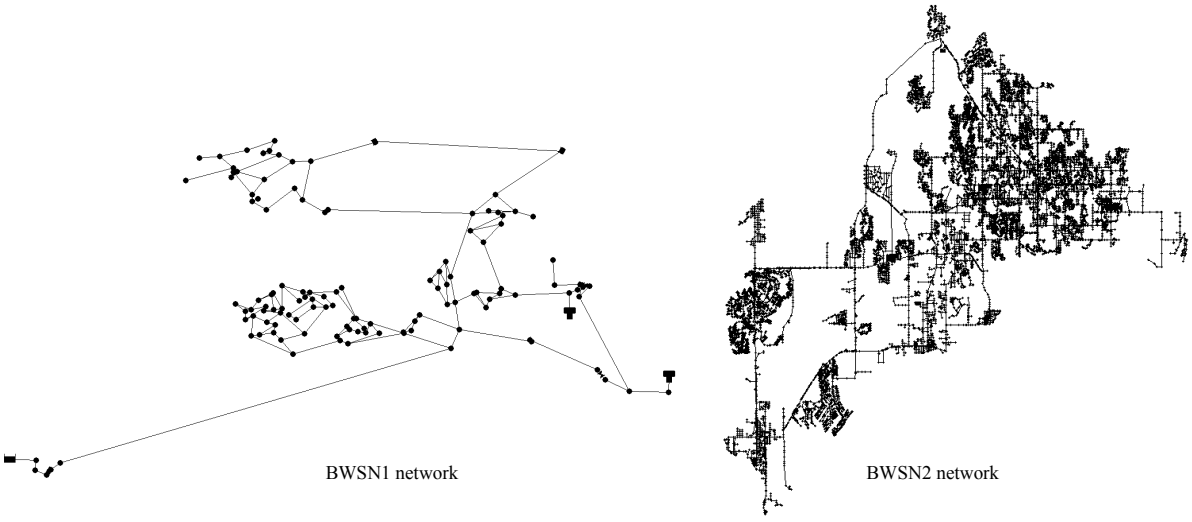

Fig. 1. Topology of the two networks of the battle of the water sensor networks (BWSN) (Ostfeld et al., 2008).

The problem data is the matrix G. For each pair of sensor m and event E_k, the entry g_{mk} can take two types of values: i) a nonnegative number that represents the consumption of the contaminated water (gallons) before sensor m detects event E_k, as described in Section III, ii) "-1", meaning that that sensor *does not detect* the event at all. The performance metric for any sensor placement solution consists of two numbers:

- The average consumption of the contaminated water for all *detected* events: g^{avg}:
$$g^{avg} = \frac{\sum_{E_k \in \mathcal{E}} I\left(\min_{m \in \mathcal{M}} g_{mk} \geq 0\right) \cdot \min_{m \in \mathcal{M}} g_{mk}}{\sum_{E_k \in \mathcal{E}} I\left(\min_{m \in \mathcal{M}} g_{mk} \geq 0\right)}, \tag{16}$$
where $I\left(\min_{m \in \mathcal{M}} g_{mk} \geq 0\right)$ is the indicator function on whether event E_k is detected at all.
- The event detection likelihood, i.e., the number of events being detected by at least one sensor, divided by the number of all events: $\mathbb{P}(\text{detect})$:
$$\mathbb{P}(\text{detect}) = \frac{\sum_{E_k \in \mathcal{E}} I\left(\min_{m \in \mathcal{M}} g_{mk} \geq 0\right)}{K}. \tag{17}$$

Clearly, these two objectives must be considered *simultaneously*. For example, one can have close to zero detection likelihood but a small average consumption of the contaminated water among the few detected events; this would not be a desirable result. The problem is thus a bi-criterion optimization, and there is a Pareto optimal boundary of $(\mathbb{P}(\text{detect}), p^{avg})$.

Our problem formulation (cf. Section III) is minimizing the average consumption of the contaminated water among *all events, detected and undetected*. Thus, we need to employ a number value in terms of the consumption of the contaminated water for estimating the impact of *undetected events*. A natural choice of this number value is provided later in Section V-C, in which we will define a "ceiling" denoted by $\bar{p}$. Our formulation can then be understood as a "scalarization" of this bi-criterion optimization problem considered in BWSN, as it scalarizes optimizing two objectives in a program with a single objective (Boyd and Vandenberghe (2004), Section 4.7.4).

B. Solutions and Evaluation Results

We now demonstrate our findings for the two BWSN benchmark networks. We present in Tables I, II III and IV the $5, 5, 4$ and 6 sets of representative sensor locations found based on the proposed algorithms (and a technique of sampling subsets of events to be described later in Section V-D) for the 129-node and 12527-node networks, each with either 5 or 20 sensors as in the BWSN competition. The resulting average consumption of the contaminated water

and detection likelihood for each set of sensor locations are also given. These results are then compared with all the previous results presented in the BWSN competition representing the state of the art, as shown in Figures 2, 3, 4 and 5. As higher detection likelihood and lower average consumption of the contaminated water are preferred, points to the right and lower in the plots represent more desirable performance.

TABLE I

OPTIMIZED SENSOR LOCATIONS, 129-NODE NETWORK, 5 SENSORS

Sets	Sensor locations	g^{avg}	$\mathbb{P}$(detect)
1^{st} set	18, 50, 69, 84, 99	1830	0.6839
2^{nd} set	11, 22, 69, 84, 116	2956	0.7286
3^{rd} set	11, 18, 32, 84, 101	3682	0.7519
4^{th} set	11, 18, 32, 84, 121	3951	0.7624
5^{th} set	11, 18, 46, 84, 121	7841	0.8004

TABLE II

OPTIMIZED SENSOR LOCATIONS, 129-NODE NETWORK, 20 SENSORS

Sets	Sensor locations	g^{avg}	$\mathbb{P}$(detect)
1^{st} set	5 , 11, 18, 19, 22, 26, 31, 32, 35, 38, 43, 47, 69, 78, 83, 84, 91, 100, 103, 120	363	0.8070
2^{nd} set	5, 11, 18, 25, 31, 32, 35, 36, 47, 69, 78, 83, 84, 91, 99, 101, 103, 104, 115, 122	405	0.8443
3^{rd} set	6, 11, 18, 22, 25, 32, 35, 36, 46, 69, 78, 83, 84, 91, 99, 101, 103, 117, 121, 123	441	0.8675
4^{th} set	11, 18, 22, 28, 32, 35, 36, 46, 69, 77, 83, 84, 91, 99, 101, 103, 113, 117, 122, 123	540	0.8830
5^{th} set	11, 18, 20, 22, 32, 35, 36, 46, 69, 75, 81, 84, 98, 101, 103, 113, 117, 122, 123, 125	678	0.8981

TABLE III

OPTIMIZED SENSOR LOCATIONS, 12527-NODE NETWORK, 5 SENSORS

Sets	Sensor locations	g^{avg}	$\mathbb{P}$(detect)
1^{st} set	3358, 3771, 10875, 11185, 11305	1830	0.2556
2^{nd} set	1480, 3358, 3771, 10875, 11305	2956	0.2794
3^{rd} set	637, 1487, 3230, 3771, 11305	3682	0.3048
4^{th} set	637, 1487, 3230, 3771, 3837	3951	0.3156

TABLE IV

OPTIMIZED SENSOR LOCATIONS, 12527-NODE NETWORK, 20 SENSORS

Sets	Sensor locations	g^{avg}	$\mathbb{P}$(detect)
1^{st} set	637, 1918, 2531, 3230, 3358, 3585, 3771, 4033, 4116, 4137, 4241, 5115, 6584, 8845, 9000, 9143, 9723, 10477, 10615, 11178	21512	0.3746
2^{nd} set	74, 1905, 1918, 2531, 3358, 3627, 3771, 3782, 4116, 4133, 4241, 4553, 6584, 7933, 8847, 9000, 9143, 9365, 10615, 11272	25798	0.3924
3^{rd} set	123, 1905, 1918, 2531, 3358, 3636, 3771, 3837, 4116, 4133, 4241, 6584, 7665, 7933, 8845, 9000, 9143, 9365, 11178, 11305	28007	0.3986
4^{th} set	1487, 1905, 2593, 3236, 3358, 3676, 3771, 3782, 4033, 4150, 4241, 4307, 7665, 8835, 9000, 9143, 9365, 10875, 11178, 11305	32330	0.4072
5^{th} set	74, 619, 1487, 1905, 2074, 2593, 3125, 3236, 3358, 3676, 3771, 3780, 4241, 7437, 7665, 9365, 10408, 10615, 10875, 11185	38059	0.4174
6^{th} set	552, 1487, 2074, 2593, 3125, 3236, 3358, 3679, 3771, 3785, 4241, 4278, 5329, 7437, 7665, 8090, 9365, 10875, 11185, 11305	45708	0.4276

We observe that our results are very competitive, and in fact provide significant performance enhancement over the state of the art. In Figure 2 and 3, for each of all sets of sensor locations in the BWSN competition, there is always one set of locations obtained by our algorithms that perform *strictly better* than it, i.e., with higher detection likelihood and lower average consumption of the contaminated water. In Figure 4 and 5, for each of all but one sets of sensor locations in the BWSN competition, there is one set of locations obtained by our algorithms that perform strictly better. Considering the overwhelming complexity of the sensor placement problems,

$$\begin{aligned} &\binom{129}{5} = 2.752 \times 10^{8}, \ \binom{129}{20} = 1.416 \times 10^{23}, \\ &\binom{12527}{5} = 2.569 \times 10^{18}, \ \binom{12527}{20} = 3.666 \times 10^{63}, \end{aligned} \tag{18}$$

our methodology shows very powerful performance in optimizing the sensor locations.

In the remainder of this section, we present details of how these results are achieved based on our methodology.

C. Algorithm Parameters

In the approximation step (4), the $\min$ function (LHS) is smoothed by a $\log\text{-sum-exp}$ function (RHS). The greater the parameter β is, the closer this approximation is, and as β goes to zero,

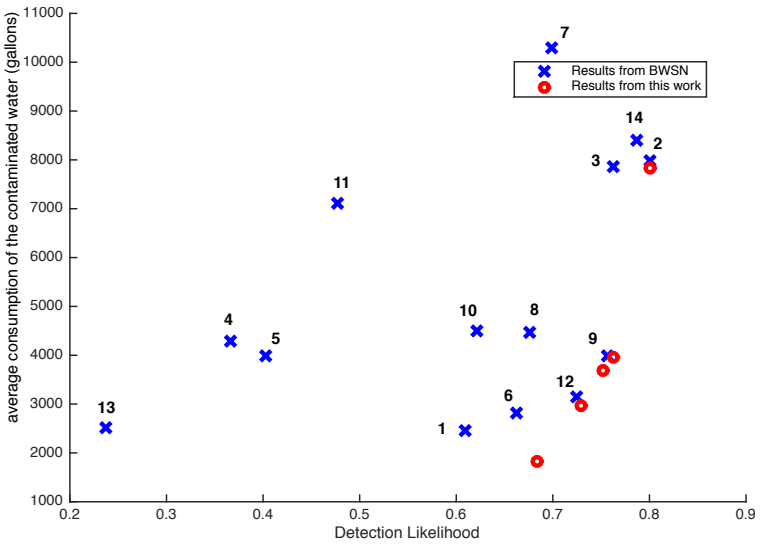

Fig. 2. Detection likelihood and average consumption of the contaminated water with 5 sensors in the 129-Node Network. BWSN labels: 1 = Berry et al.; 2 = Dorini et al.; 3 = Eliades and Polycarpou; 4 = Ghimire and Barkdoll (demand); 5 = Ghimire and Barkdoll (mass); 6 = Guan et al.; 7 = Gueli; 8 = Huang et al.; 9 = Krause et al.; 10 = Ostfeld and Salomons; 11 = Preis and Ostfeld; 12 = Propato and Piller; 13 = Trachtman; 14 = Wu and Walski.

the RHS of (4) converges to the LHS. However, β cannot be set arbitrarily small due to the following numerical reason. Specifically, if g_{mk} is large but β is relatively small, $\exp\left(-\frac{g_{mk}}{\beta}\right)$ in (4) can fall below machine precision and be rounded to *zero*. For an event E_k, if g_{mk} is too large (compared to β) for every $m \in \mathcal{M}$, $\sum_{m\in\mathcal{M}} \exp\left(-\frac{g_{mk}}{\beta}\right)$ and $\log\left(\sum_{m\in\mathcal{M}} \exp\left(-\frac{g_{mk}}{\beta}\right)\right)$ will be computed as 0 and $-\infty$, respectively. *This will then lead to a* $-\infty$ *objective in* (7). This situation can in fact happen very frequently, or even at all times, when selecting just a small number of sensors but evaluating a large number of events, as considered in the BWSN benchmarks. This is because, it can happens that, no matter where the sensors are located, there is always some "worst-case" event (among all) that is poorly detected and leads to large consumption of the contaminated water, driving $\exp\left(-\frac{g_{mk}}{\beta}\right)$ to zero.

To overcome this issue, for any chosen β, we impose a ceiling $\bar{g}$ on the entries in G: $g_{mk} \leftarrow \min\left(g_{mk}, \bar{g}\right), \forall m, k$, where $\bar{g}$ is set so that $\exp\left(-\frac{\bar{g}}{\beta}\right)$ is above machine precision. Clearly, both parameters β and $\bar{g}$ affect the approximation of the original problem, which becomes more precise with a smaller β and a larger $\bar{g}$. From the above, however, the ratio $\frac{\bar{g}}{\beta}$ cannot exceed a level determined by the machine precision. There is thus a trade off between the choices of β and $\bar{g}$. With a machine precision around 5×10^{-324} (as on the laptop used for most of the

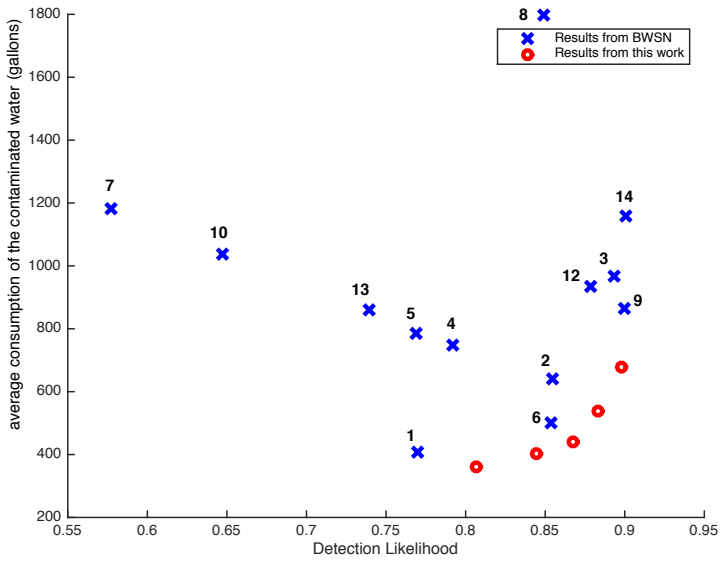

Fig. 3. Detection likelihood and average consumption of the contaminated water with 20 sensors in the 129-Node Network. BWSN labels: 1 = Berry et al.; 2 = Dorini et al.; 3 = Eliades and Polycarpou; 4 = Ghimire and Barkdoll (demand); 5 = Ghimire and Barkdoll (mass); 6 = Guan et al.; 7 = Gueli; 8 = Huang et al.; 9 = Krause et al.; 10 = Ostfeld and Salomons; 11 = Preis and Ostfeld; 12 = Propato and Piller; 13 = Trachtman; 14 = Wu and Walski.

simulations in this section), $\frac{\bar{g}}{\beta}$ cannot exceed $-\log(5 \times 10^{-324}) = 744.44$.

With this limit in mind, the choice of β (and correspondingly $\bar{g}$) depends on a rough knowledge of the problem data G: in particular, we make a rough guess on the *ballpark of the optimal objective value of the original problem* (2)*, namely the minimum achievable average consumption of the contaminated water*. The idea is that, *as long as β is somewhat small relative to this value, it needs not be too small*. This ballpark value can be obtained by simply evaluating any sensor placement heuristics at hand. As we employ the BWSN benchmarks in this paper, we take the best results achieved in the BWSN competition as indicators of this ballpark value. Specifically,

- For selecting 5 sensor locations in the 129-node network, the lowest average consumption of the contaminated water achieved in the BWSN competition is 2459 gallons. Accordingly, we choose $\beta = 1000$.
- For selecting 20 sensor locations in the 129-node network, the lowest average consumption of the contaminated water achieved in the BWSN competition is 408 gallons. Accordingly, we choose $\beta = 200$.
- For selecting 5 sensor locations in the 12527-node network, the lowest average consumption

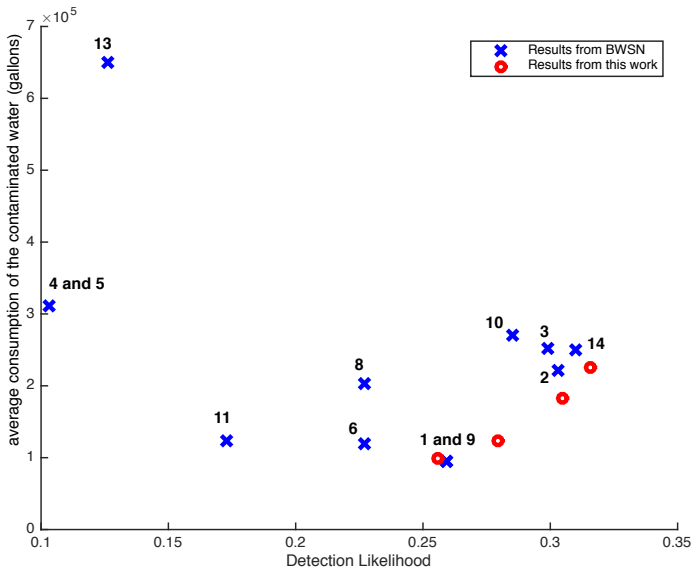

Fig. 4. Detection likelihood and average consumption of the contaminated water with 5 sensors in the 12527-Node Network. BWSN labels: 1 = Berry et al.; 2 = Dorini et al.; 3 = Eliades and Polycarpou; 4 = Ghimire and Barkdoll (demand); 5 = Ghimire and Barkdoll (mass); 6 = Guan et al.; 7 = Gueli; 8 = Huang et al.; 9 = Krause et al.; 10 = Ostfeld and Salomons; 11 = Preis and Ostfeld; 12 = Propato and Piller; 13 = Trachtman; 14 = Wu and Walski.

of the contaminated water achieved in the BWSN competition is 95403 gallons. Accordingly, we choose $\beta = 40000$.

- For selecting 20 sensor locations in the 12527-node network, the lowest average consumption of the contaminated water achieved in the BWSN competition is 17456 gallons. Accordingly, we choose $\beta = 10000$.

D. Random Subsets of Events

The problem sizes for both BWSN benchmarks are exceedingly large, as we have more than 37000 events for the 129-node network, and more than 25000 events for the 12527-node network. To speed up the proposed algorithms, we employ the following technique: Instead of running the greedy and branch and bound algorithms for the entire set of events, we *randomly sample subsets of the events* for running the algorithms. We then repeat this random sampling for a considerable number of trials, and pick the best achieved results. Interestingly, repeatedly sampling a very small number of events, on the order of 20 to 200, can already lead to great performance (as demonstrated in the evaluation results above).

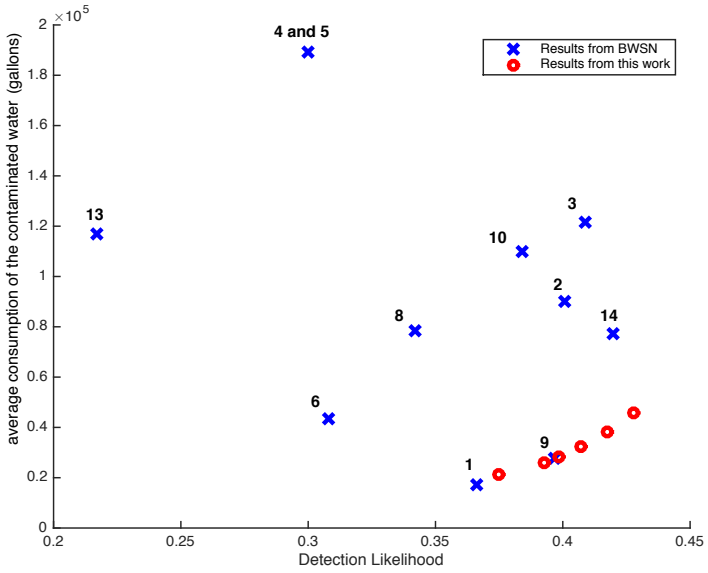

Fig. 5. Detection likelihood and average consumption of the contaminated water with 20 sensors in the 12527-Node Network. BWSN labels: 1 = Berry et al.; 2 = Dorini et al.; 3 = Eliades and Polycarpou; 4 = Ghimire and Barkdoll (demand); 5 = Ghimire and Barkdoll (mass); 6 = Guan et al.; 7 = Gueli; 8 = Huang et al.; 9 = Krause et al.; 10 = Ostfeld and Salomons; 11 = Preis and Ostfeld; 12 = Propato and Piller; 13 = Trachtman; 14 = Wu and Walski.

Moreover, randomly selecting subsets of events also allows us to explore the *Pareto optimal boundary* (the lower-right frontier in Figures 2, 3, 4 and 5) of average consumption of the contaminated water vs. detection likelihood. Note that, for each fixed set of parameters and subset of events, our algorithm runs deterministically, and the optimal value is unique. The vastly different random subsets of events provide a diversity in the input data for exploring different points on the Pareto optimal boundary. The choice of the ceiling parameter $\bar{g}$ also provides a degree of freedom for exploring different points on the Pareto optimal boundary.

E. Power of Greedy Heuristic

Surprisingly, we found that, with the above parameter choices and random sampling of small subsets of events, the proposed greedy heuristic can already perform extremely well. In fact, all the results shown in Section V-B are achieved simply by greedy heuristic applied to different subsets of events (but of course evaluated using the entire set of events). Moreover, we used the proposed branch and bound algorithm beyond the greedy heuristic for the 129-node network, and observed the following interesting fact: For the cases that generate the solutions in Table I, although it took a few hundreds of iterations for the upper bound and lower bound to meet,

i.e., proving the global optimality of the solution, *the optimal solution is always found in just 1 iteration, i.e., by the greedy heuristic*. In other words, while i_{prove} (defined at the end of Section IV-B) can be a few hundred, i_{achieve} is just 1. A representative plot for the progression of the lower and upper bounds is given in Figure 6.

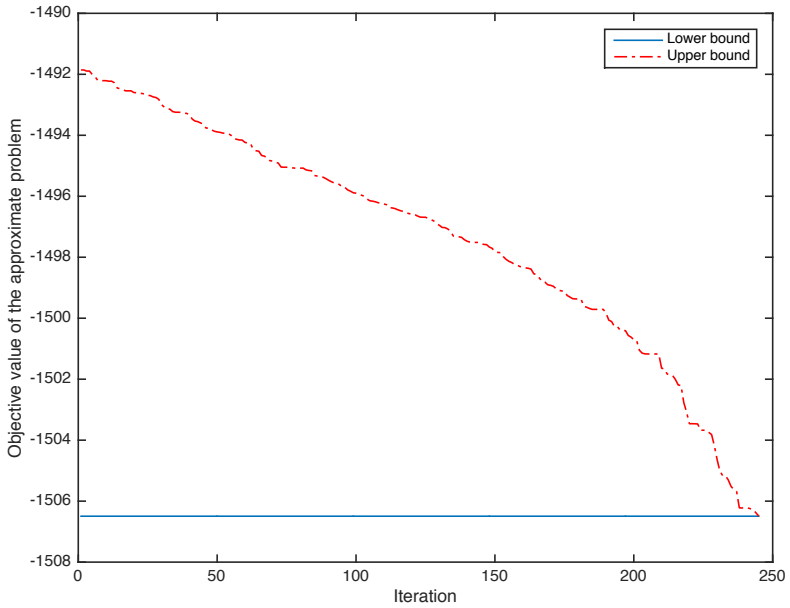

Fig. 6. Progression of the lower and upper bounds with the branch and bound algorithm.

This discovered power of the proposed greedy heuristic is greatly useful in practice, particularly for networks of *very large sizes* such as the 12527-node benchmark network. For networks of such sizes, continuous optimization based approaches can take exceedingly long times to finish, and the proposed simple but powerful greedy heuristic becomes very appealing.

VI. Conclusions

We have studied optimal sensor placement in water distribution systems for detecting contamination events and reducing consumption of the contaminated water. We employ two benchmark networks of 129 nodes and 12527 nodes and the corresponding contamination events from the Battle of Water Sensor Networks for performance evaluation and comparison to the state of the art. We employ the performance metric determined by the average consumption of the contaminated water among all events, and optimize the sensor placement to minimize this metric. Novel modeling and approximation techniques are developed to transform the combinatorial

optimization of sensor placement into a mixed integer-convex programming problem. A greedy heuristic is first developed. We have then developed a branch and bound algorithm to find the global optimum of the MICP, based on convex relaxation and the greedy heuristic. The branch and bound algorithm allows us to keep improving sensor placement solutions as long as run-time constraints permit, and provides bounds that indicate the gap to the global optimum. We have further developed a technique of repeatedly and randomly sampling subsets of events for running the algorithms. It has been demonstrated that significant performance enhancement of the state of the art is achieved by the proposed algorithms, in particular by the greedy heuristic applied to repeatedly sampled random subsets of events. The developed methodology is broadly applicable to employing other metrics for optimizing water sensor placement. In general, to achieve different points on the Pareto optimal boundary under different metrics, tuning the algorithm parameters will play an important role, and a systematic way of parameter tuning is left for future research.

VII. Acknowledgements

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