Solar Energy Sharing in Net Metered Community Microgrids: Can the Social Goals be Achieved?

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***Abstract*—Solar energy sharing among prosumers via peer-to-peer energy trading in a net metered community microgrid is studied. The trading interactions among the prosumers are analyzed as a market with transferable payoff. The outcome of the prosumer's trading interactions is predicted by the competitive equilibrium of the market, for which a closed form expression is derived. The competitive equilibrium also offers a stable/in the core cost allocation mechanism for the energy-sharing prosumers in an aggregated microgrid. The competitive equilibrium reveals that, in a net metered microgrid with a lower to medium level of solar penetration, all the economic benefit from solar energy sharing goes to the prosumers who have solar panels. In other words, even though a key social incentive for solar energy sharing is to let consumers without solar panels to have access to locally generated solar power from their neighbors who have solar panels, their economic benefits from doing so are however fundamentally limited.**

I. Introduction

There has been a continuing proliferation of rooftop solar panels in power distribution systems around the world. Such rooftop solar resources supply green energy to the consumers directly, reducing the amount of energy they draw from the grid and hence their bills from the utility. When a rooftop solar panel produces more energy than the consumer's load, the excess energy is typically fed back to the grid, making the consumer a "prosumer". To account for rooftop solar energy fed back to the grid, a popular scheme is "net metering" [1]. For example, a prosumer may accumulate credits for all the excess energy fed back to the grid over a year, and may exchange them for a monetary payoff from the utility. Other forms of net metering exist with similar spirits but different technical details.

Recently, there have been increasing voices and support for allowing rooftop solar energy generated in local communities to be shared via, e.g., peer-to-peer trading, within the communities, (see, e.g., "community solar" [2]). Instead of each customer interfacing with the utility separately, such local solar energy sharing schemes would allow customers within a community to exchange energies internally, thus reducing their collective energy and information exchanges with the utility. With local solar energy sharing, a) there are potentials for both prosumers (who can generate excess solar power) and consumers (who can consume these power) in a community to benefit economically, and b) both producers and consumers of shared solar power can get credits for the generation and use of green energy, *locally*. In particular, consumers who do not have solar panel may now have access to possibly cheaper solar power generated by their neighbors in the same community. There is a hope of achieving "energy democratization" as a social goal [3].

Notably, among technologies that enable and catalyze community solar energy sharing are the "blockchain" technologies [4]. Blockchains allow distributed, safe, and transparent transactions for solar energy trading, and have already had mature implementations in various places of the world (see e.g. Brooklyn Microgrid among others) [5], [6], [7], [8]. There have also been campus demonstrations of solar energy trading with blockchains enabled auctions [9]. Furthermore, solar energy trading in community microgrids is a concrete example of transactive energy systems in general [10].

In this paper, we study what happens when prosumers in a net metered community microgrid trade solar energies in a transparent and peer-to-peer fashion, enabled by technologies such as blockchains. We show that such solar energy trading can be analyzed as a market with transferable payoff [11]. We then derive a simple closed form expression of the *competitive equilibrium* (CE) of this market, which offers a prediction of the outcome of solar energy trading within the community. In addition, the CE also offers a stable/in the core cost allocation solution to the prosumers when they aggregate together as one jointly net metered entity. While energy democratization is a key social goal, however, the CE reveals that, with a lower to medium solar penetration level, all the economic benefit from solar energy trading goes to prosumers with solar panels who are net producers. Thus, consumers without solar panel cannot benefit economically from transparent and hence competitive solar energy trading. This outcome nonetheless provides strong incentives for consumers to invest in rooftop solar panels, until the solar penetration level of the entire community rises to the point where the aggregated community has a significant probability of becoming a net energy producer.

The remainder of the paper is organized as follows. In Section II, we establish the models for net metering of prosumers, solar energy trading, and solar energy aggregation. In Section III, we formulate solar energy trading as a market with transferable payoff, and derive the competitive equilibrium of the market as well as a stable/in the core cost allocation for an aggregated community microgrid. In Section IV, we discuss the implications of the CE on different types of prosumers, and on the social goals of solar energy sharing. Conclusions are drawn in Section V.

II. System Model

A. Net metering in a microgrid

We consider a microgrid with N energy prosumers, denoted by $\mathcal{N} = \{1, \dots, N\}$, each with possibly a solar energy generator of its own (e.g., a rooftop solar panel). A special case of a prosumer is an energy consumer who has no solar energy generator. A net-metered prosumer is typically billed according to the following rule:

$$C_i = p^b(l_i - x_i)_+ - p^s(x_i - l_i)_+, \tag{1}$$

where a) C_i is the cost of prosumer i, i.e., its payment made to the utility, b) l_i and x_i are the load and solar generation of prosumer i, respectively, c) p^b is the regular price of energy if a prosumer draws power from the grid to supply its (net) load, d) p^s is the price of energy sold back to the grid if a prosumer has excess generation after fulfilling its own load, and e) the notation $(\cdot)_+$ represents $\max\{\cdot, 0\}$.

In other words,

- If a prosumer has a positive net load, i.e., $l_i > x_i$, it buys energy to supply its net load $l_i - x_i$ at the regular energy price p^b.
- If a prosumer has a negative net load, i.e, $l_i < x_i$, it sells the excess energy $x_i - l_i$ at a (typically discounted) energy price $p^s(\leq p^b)$.

We note that, in practice, a prosumer can be compensated for its excess energy $(x_i - l_i)_+$ via a variety of ways of net metering, such as accumulating credits in its account. The above model (1) serves as a useful abstraction of these net metering mechanisms.

Since all the prosumers face the same prices p^b and p^s, we define

$$C(x, l) \triangleq p^b(l - x)_+ - p^s(x - l)_+, \tag{2}$$

and accordingly, $C_i = C(x_i, l_i)$.

B. Solar energy sharing via peer-to-peer trading

There has been a recent surge of interest in allowing prosumers in the same community microgrid to trade their (excess) solar energy among each other in a peer-to-peer fashion: Any pair of prosumers i and j may agree to a trade in which prosumer j sells an amount of (excess) solar energy x_{ji} to prosumer i at a price of p_{ji} for each unit of power. Such peer-to-peer trades enable prosumers to share solar energy with each other, in particular those who have solar panels with those who do not [12].

Notably, modern technologies such as blockchains become key enablers of such peer-to-peer energy trades, and moreover create a *transparent* and hence *competitive* environment for such trades [4]. In such competitive environments, it is intuitive that the prices $\{p_{ji}, \forall i, j\}$ will converge to a *single* price, in particular because energy is a *single commodity* regardless of its source of generation. Indeed, in principle, a unit of energy from prosumer j vs. prosumer $k \neq j$ would not make a difference to prosumer i. The outcome of such competitive environments will be analyzed in a rigorous framework in Section III.

C. Cost allocation for prosumers in an aggregated microgrid

A related perspective of analyzing the effect of net metering on a microgrid is cost allocation within an aggregation of prosumers [13], [14], [15]. Notably, the *prosumers always benefit from aggregating together as one entity* to interface with the utility. More generally, consider any subset of prosumers $\mathcal{S} \subseteq \mathcal{N}$: if they join together as one aggregate prosumer and interface with the utility via net metering, their total payment would be

$$C_{\mathcal{S}} = C(x_{\mathcal{S}}, l_{\mathcal{S}}) = p^b(l_{\mathcal{S}} - x_{\mathcal{S}})_+ - p^s(x_{\mathcal{S}} - l_{\mathcal{S}})_+, \tag{3}$$

where $x_{\mathcal{S}} \triangleq \sum_{i \in \mathcal{S}} x_i$, $l_{\mathcal{S}} \triangleq \sum_{i \in \mathcal{S}} l_i$. It is immediate to verify that aggregation of prosumers always leads to a no higher total cost, i.e., $\forall \mathcal{S} \subseteq \mathcal{N}, \mathcal{T} \subseteq \mathcal{N}, \mathcal{S} \cap \mathcal{N} = \emptyset$, we have

$$\begin{aligned} C_{\mathcal{S} \cup \mathcal{T}} &= C(x_{\mathcal{S} \cup \mathcal{T}}, l_{\mathcal{S} \cup \mathcal{T}}) \\ &\leq C_{\mathcal{S}} + C_{\mathcal{T}} = C(x_{\mathcal{S}}, l_{\mathcal{S}}) + C(x_{\mathcal{T}}, l_{\mathcal{T}}). \end{aligned} \tag{4}$$

As a result, aggregation of *all* the prosumers in a microgrid achieves the minimum possible total cost:

$$C_{\mathcal{N}} = C(x_{\mathcal{N}}, l_{\mathcal{N}}) = p^b(l_{\mathcal{N}} - x_{\mathcal{N}})_+ - p^s(x_{\mathcal{N}} - l_{\mathcal{N}})_+. \tag{5}$$

A natural question that arises is thus *how should the minimum total cost* (5) *be allocated to each of all the prosumers?* We will show in Section III-C that the competitive equilibrium for the peer-to-peer trading scenario provides a good answer to this question.

III. Competitive Equilibrium of Peer-to-Peer Solar Energy Trading

With prosumers trading solar energies among themselves, there is only one type of "goods" being traded around — the solar energy. Thus, the cost of a prosumer at the end of all the trades depends on a) the amount of solar energy this prosumer ends up with, and b) the monetary payments associated with the solar energy trades:

$$\tilde{C}_i = C(z_i, l_i) + p \cdot (z_i - x_i), \tag{6}$$

where z_i is the amount of solar energy prosumer i ends up with after all the trades, and $p \cdot (z_i - x_i)$ is its cost of buying an amount of energy $z_i - x_i$ (can be negative). As explained in Section II-B, (6) assumes that all the solar energy trades are at the same per unit price p due to the transparent and hence competitive nature of the trades.

In this section, we first formalize solar energy trading among prosumers as a market with transferable payoff, and then develop its competitive equilibrium (CE) as the predicted outcome from solar energy trading. We next show that the CE also offers a stable/in the core cost allocation among aggregated prosumers in a net metered microgrid.

A. Market with transferable payoff

We define the following market with transferable payoff [11]:

- The prosumers, denoted by $\mathcal{N}$, are a finite set of N agents.
- There is one type of input goods — solar energy.

- Each agent $i \in \mathcal{N}$ has an "endowment" in the amount of $x_i \in \mathbb{R}_+$ — the solar energy of prosumer i.
- Each agent $i \in \mathcal{N}$ has a continuous, nondecreasing, and concave "production" function $f_i : \mathbb{R}_+ \to \mathbb{R}$:

$$f_i(x_i) = -C(x_i, l_i) \quad (7)$$
$$= p^s(x_i - l_i)_+ - p^b(l_i - x_i)_+. \quad (8)$$

Since all the "production" functions $\{f_i\}$ produce the same type of transferable output, i.e., monetary payoff, the above formulation defines a market with transferable payoff.

B. Competitive equilibrium

For the above market with transferable payoff, a competitive equilibrium [11] is defined as a price-quantity pair of $p^* \in \mathbb{R}_+$ and $\boldsymbol{z}^* \in \mathbb{R}_+^N$, such that,

i) For each agent i, z_i^* solves the following problem:

$$\max_{z_i \in \mathbb{R}_+} \left(f_i\left(z_i\right) - p^*\left(z_i - x_i\right)\right). \quad (9)$$

ii) $\boldsymbol{z}^*$ is a redistribution, i.e., $\sum_{i\in\mathcal{N}} z_i^* = \sum_{i\in\mathcal{N}} x_i$.

The intuition of a CE is the following: At the price p^*, i) to maximize its payoff (i.e., minimize its cost (6)), each agent i can trade *any* amount of the input (solar energy) on the market *without* worrying whether there is enough supply or demand to fulfill its trade request, and ii) collectively, the market of input supply and demand *still clears*, i.e., the resulting $\boldsymbol{z}^*$ from the optimal trades is feasible.

At a competitive equilibrium $(p^*, \boldsymbol{z}^*)$, p^* is called the *competitive price*, and the value of the maximum of (9) is called the *competitive payoff* of agent i.

For this market with transferable payoff, we now have the following theorem in deriving its unique CE, whose proof is relegated to Appendix A:

Theorem 1: Competitive equilibrium exists, and the competitive payoffs necessarily take the following form: $\forall i \in \mathcal{N}$,

$$f_i^* = \begin{cases} -p^b\left(l_i - x_i\right), \text{ if } x_{\mathcal{N}} - l_{\mathcal{N}} < 0 \\ -p^s\left(l_i - x_i\right), \text{ if } x_{\mathcal{N}} - l_{\mathcal{N}} > 0 \\ -p^*\left(l_i - x_i\right), \text{ if } x_{\mathcal{N}} - l_{\mathcal{N}} = 0 \end{cases}, \quad (10)$$

where $p^s \le p^* \le p^b$, and p^* can be chosen arbitrarily within this range.

Accordingly, the cost of prosumer i at the CE, denoted by $\tilde{C}_i^*$, equals $-f_i^*$. In other words, with competitive peer-to-peer solar energy trades, at equilibrium, each prosumer i's cost will be $\tilde{C}_i^* = -f_i^*$, (in comparison to C_i in (1) without trades). Later in Section IV, we will provide a detailed discussion on the implications of this CE (10).

C. Stable cost allocation for aggregated prosumers

As discussed in Section II-C, the minimum total cost of the prosumers in a microgrid is achieved when all of them aggregate as one entity net metered by the utility (cf. (5)). To split this minimum cost among the prosumers, it is desirable for the cost allocation $\{\tilde{C}_i\}$ to satisfy the following conditions. (With a slight abuse of notation, we have reused the notation $\tilde{C}_i$ as in (6), which indeed represents a cost allocation solution).

- *Budget balance*:

$$\sum_{i\in\mathcal{N}} \tilde{C}_i = C_{\mathcal{N}}, \quad (11)$$

where $C_{\mathcal{N}}$ is defined in (5).
- *Stability*: For *all* subsets of prosumers $\mathcal{S} \subseteq \mathcal{N}$,

$$\sum_{i\in\mathcal{S}} \tilde{C}_i \le C_{\mathcal{S}}, \quad (12)$$

where $C_{\mathcal{S}}$ is defined in (3), which is the total cost of the *subset* of prosumers $\mathcal{S}$ had they joined as one entity net metered by the utility. In other words, no subset of prosumers can possibly enjoy a lower cost by leaving the full aggregation of $\mathcal{N}$.

We note that such a stability condition is also termed "in the core" of a coalitional game defined with $\{C_{\mathcal{S}}, \forall \mathcal{S} \subseteq \mathcal{N}\}$ [11]. Next, we will show that the market with transferable payoff defined above leads to the same coalitional game.

Specifically, for any coalition of a subset of prosumers $\mathcal{S} \subseteq \mathcal{N}$, define

$$v\left(\mathcal{S}\right) = \max_{\{z_i \in \mathbb{R}_+, i\in\mathcal{S}\}} \sum_{i\in\mathcal{S}} f_i(z_i) \quad (13)$$
$$s.t. \sum_{i\in\mathcal{S}} z_i = \sum_{i\in\mathcal{S}} x_i.$$

In other words, $\{z_i, i \in \mathcal{S}\}$ denotes a *redistribution* of the total solar power $\sum_{i\in\mathcal{S}} x_i$ among the members of $\mathcal{S}$. This $v(\mathcal{S})$ represents the *maximum* total payoff that the members of $\mathcal{S}$ can achieve among all possible redistributions, computed according to f_i defined in (7). The core of this coalitional game is also called the "core of the market".

We now have the following lemma on the equivalence of this coalitional game defined with (13) to that with $\{C_{\mathcal{S}}, \forall \mathcal{S} \subseteq \mathcal{N}\}$. The proof is relegated to Appendix B.

Lemma 1: The values of coalitions (13) are the same as the negative of their costs (3):

$$v(\mathcal{S}) = -C_{\mathcal{S}}. \quad (14)$$

As a result, from the properties of market with transferable payoff (cf. Propositions 264.2 and 267.1 in [11]), we have that the CE of the market of transferable payoff provides a cost allocation solution in the core.

Corollary 1: This coaltional game has a non-empty core, and a stable/in the core cost allocation (cf. (11) and (12)) is given by the negative of the competitive payoffs of the market (cf. (10)),

$$\tilde{C}_i^* = -f_i^* = \begin{cases} p^b\left(l_i - x_i\right), \text{ if } x_{\mathcal{N}} - l_{\mathcal{N}} < 0 \\ p^s\left(l_i - x_i\right), \text{ if } x_{\mathcal{N}} - l_{\mathcal{N}} > 0 \\ p^*\left(l_i - x_i\right), \text{ if } x_{\mathcal{N}} - l_{\mathcal{N}} = 0 \end{cases}, \quad (15)$$

where $p^s \le p^* \le p^b$, and p^* can be chosen arbitrarily within this range.

IV. Impact of the competitive equilibrium on the social goals of solar energy sharing

We now analyze the implications of the competitive equilibrium of solar energy sharing. From (10) and (15), at the CE,

- If the total solar energy generated in the entire microgrid is *less* than the total load therein, i.e., $x_{\mathcal{N}} - l_{\mathcal{N}} < 0$,
  - For any prosumer i with a positive net load, i.e., $l_i > x_i$, it has the *same* cost of purchasing energy as if it were separately net metered (cf. (1)):
  $$\tilde{C}_i^* = C_i = p^b(l_i - x_i). \tag{16}$$
  - For any prosumer j with a positive net generation, i.e., $x_j > l_j$, it earns a higher profit from selling excess solar generation than when it is separately net metered:
  $$-\tilde{C}_j^* = p^b(x_j - l_j) > -C_j = p^s(x_j - l_j), \tag{17}$$
  assuming $p^s < p^b$.
- If the total solar energy generated in the entire microgrid is *more* than the total load therein, i.e., $x_{\mathcal{N}} - l_{\mathcal{N}} > 0$,
  - For any prosumer i with a positive net load, i.e., $l_i > x_i$, it has a lower cost of purchasing energy than when it is separately net metered (cf. (1)):
  $$\tilde{C}_i^* = p^s(l_i - x_i) < C_i = p^b(l_i - x_i), \tag{18}$$
  assuming $p^s < p^b$.
  - For any prosumer j with a positive net generation, i.e., $x_j > l_j$, it earns the same profit from selling excess solar generation as if it were separately net metered:
  $$-\tilde{C}_j^* = -C_j = p^s(x_j - l_j). \tag{19}$$

A. Who benefits from solar energy trading/sharing?

From the above analysis, when the entire microgrid has a positive total net load, *all* the economic benefit from solar energy trading/sharing goes to those prosumers with a positive net generation; otherwise, all the benefit goes to those with a positive net load.

Notably, however, in community microgrids with a lower to medium level of solar penetration, *it is almost always the case that a microgrid has a positive total net load*. In this case, the CE predicts that, *even with solar energy trading/sharing in full operation, those consumers without solar generation still cannot gain any economic benefit from it*. Indeed, they would pay the *same* amount of money to supply their loads as if they purchased energy from the utility (cf. (16)).

This is unfortunately not a desired outcome, since one key social goal of enabling solar energy trading/sharing is to allow consumers without solar panels to enjoy cheaper locally generated excess solar energy by their neighbors who have solar panels. The CE reveals that solar energy trading/sharing economically *only benefits* those prosumers who have solar panels and are generating excess solar power.

An intuitive explanation of why the above happens is as follows. When there is more load than solar generation in a community microgrid, i.e., $x_{\mathcal{N}} < l_{\mathcal{N}}$, the net-consumers *compete* for the "scarce" solar generation. Because the alternative to buying local solar power is buying from the utility at a regular price p^b, as long as a net-producer offers a price slightly lower than p^b, net-consumers would have an incentive to buy from it. Because $x_{\mathcal{N}} < l_{\mathcal{N}}$, there is *more demand than supply* of solar energy (at a price cheaper than p^b). Thus, at equilibrium, the selling price of solar energy p^* will reach p^b, and all the economic benefit goes to the net-producers.

B. Incentives for prosumer participation

Another interesting implication of the competitive equilibrium is the resulting incentives for prosumer participation in solar energy sharing. In particular, consider a set of prosumers $\mathcal{N} = \{1, 2, \ldots, N\}$ already joining together for solar energy trading, and another prosumer $N+1$ who may also join in. The question is, who would prefer prosumer $N+1$ to join for solar energy trading, and who would not?

First, it is clear that having the newcomer $N+1$ join the existing aggregation $\mathcal{N}$ is preferred from both the perspectives of $\mathcal{N}$ and the newcomer $N+1$. In other words,

$$\tilde{C}_{\mathcal{N}}^* \le C_{\mathcal{N}}, \text{ and } \tilde{C}_{N+1}^* \le C_{N+1}, \tag{20}$$

where $\tilde{C}_{\mathcal{N}}^*$ is the total cost of prosumers $\mathcal{N}$ *at the new CE with prosumer* $N+1$ *joining* $\mathcal{N}$, and $\tilde{C}_{N+1}^*$ is that of prosumer $N+1$. We note that (20) is immediately implied by that CE is in the core (cf. (12) and Corollary 1).

However, inside the existing aggregation $\mathcal{N}$, the incentives on whether to welcome the newcomer $N+1$ are more complicated. To understand the incentives, we model the solar generation and consumer loads as *random variables* $\{X_i\}$ and $\{L_i\}$. Now, consider the following two types of prosumers, qualitatively defined:

- *Type 1, "Net-Consumers"*: prosumers who (almost) always have positive net loads;
- *Type 2, "Net-Producers"*: prosumers who have high probabilities of having positive net generation.

In practice, consumers who do not have rooftop solar panels fall into Type 1, and those who do typically fall into Type 2. Accordingly, we term Type 1 prosumers "Net-Consumers", and Type 2 prosumers "Net-Producers". From the CE (15) and its analysis in the previous subsection,

- When $x_{\mathcal{N}} < l_{\mathcal{N}}$, net-consumers do not benefit from solar energy sharing, whereas net-producers do.
- When $x_{\mathcal{N}} > l_{\mathcal{N}}$, net-consumers benefit from solar energy sharing, whereas net-producers do not.

As a result, *net-consumers would prefer a higher probability of the event $X_{\mathcal{N}} > L_{\mathcal{N}}$, and net-producers would prefer the opposite.* Accordingly, the attitudes of different types of prosumers toward a newcomer $N+1$ will be as follows:

- If the newcomer $N+1$ is a *consumer without a solar panel*, including it into $\mathcal{N}$ would further decrease the (typically already low) probability of the entire aggregation being a net producer, i.e.,

$$\mathbb{P}\left(X_{\mathcal{N}\cup(N+1)} > L_{\mathcal{N}\cup(N+1)}\right) < \mathbb{P}\left(X_{\mathcal{N}} > L_{\mathcal{N}}\right). \quad (21)$$

Thus, prosumer $N+1$'s participation in solar energy trading will be welcomed by net-producers, but opposed by net-consumers.

- If the newcomer $N+1$ is a *prosumer with a solar panel that generates excess solar power with a high probability*, including it into $\mathcal{N}$ would increase the probability of the entire aggregation being a net producer, i.e.,

$$\mathbb{P}\left(X_{\mathcal{N}\cup(N+1)} > L_{\mathcal{N}\cup(N+1)}\right) > \mathbb{P}\left(X_{\mathcal{N}} > L_{\mathcal{N}}\right). \quad (22)$$

Thus, prosumer $N+1$'s participation in solar energy trading will be welcomed by net-consumers, but opposed by net-producers.

Consequently, prosumers of either types (net-consumers and net-producers) are always incentivized to seek new prosumers of the *other* type to join solar energy trading, and oppose any new prosumers of its own type from joining.

One way to resolve the above tension is to ensure free entry and exit of prosumers into and out of solar energy trading. From the CE, a prosumer is always incentivized to join as opposed to leaving. With a lower to medium level of solar penetration in a community microgrid, a vast majority of prosumers are consumers without solar panel. With free entries of more net-consumers than net-producers, it will continue to be the case where the entire aggregation is almost always a net-load (i.e., $X_{\mathcal{N}} < L_{\mathcal{N}}$), and the analysis in Section IV-A continues to hold.

C. Incentives on solar energy investment

Nonetheless, the competitive equilibrium provides a strong incentive for investing on rooftop solar panels by the consumers. As long as the solar penetration level in a community is still lower to medium, the probability of the event $X_{\mathcal{N}} > L_{\mathcal{N}}$ will stay close to zero, and all the economic benefit from solar energy trading will continue to go to those who have solar panels and generate excess solar power. When the solar penetration level becomes sufficiently high so that the probability of the event $X_{\mathcal{N}} > L_{\mathcal{N}}$ becomes significant, the incentive for investing on rooftop solar may slow down.

V. Conclusion

We have studied the problem of solar energy sharing among prosumers via transparent peer-to-peer solar energy trading in a net metered community microgrid. We have shown that the problem can be analyzed as a market with transferable payoff. We have then derived the competitive equilibrium of this market, which offers a prediction of the outcome of solar energy trading among prosumers. We have shown that the CE also offers a stable/in the core cost allocation solution to an aggregated community microgrid. The CE reveals that, in a community microgrid with a lower to medium level of solar penetration, all the economic benefit from solar energy sharing goes to prosumers with solar panels who generate excess solar power, and thus consumers without solar panel cannot benefit economically from solar energy trading.

Appendix A
Proof of Theorem 1

With the production function $f_i(x_i)$ defined in (7), we observe that $f_i(x_i)$ is a *piecewise linear* function:

$$f_i'(x_i) = \begin{cases} p^b, & \text{if } x_i < l_i \\ p^s, & \text{if } x_i > l_i \end{cases}. \quad (23)$$

As a result, at a CE, we must have $p^b \le p^* \le p^s$. Otherwise, by solving (9), either all RPPs would sell all of their power, or all of them would buy an infinite amount of power; Neither case would clear the market with $\sum_{i\in\mathcal{N}} z_i^* = \sum_{i\in\mathcal{N}} x_i$.

We now analyze the optimal behavior of any agent i under the following three scenarios of the competitive price p^*:

- If $p^* = p^b$, the maximum of (9) is achieved if and only if $z_i \le l_i$.
- If $p^* = p^s$, the maximum of (9) is achieved if and only if $z_i \ge l_i$.
- If $p^s < p^* < p^b$, the maximum of (9) is achieved if and only if $z_i = l_i$.

To derive the competitive price p^* that clears the market with $\sum_{i\in\mathcal{N}} z_i^* = \sum_{i\in\mathcal{N}} x_i$, we consider the following three scenarios:

Case i) $x_{\mathcal{N}} - l_{\mathcal{N}} < 0$: As result, at the CE, $\sum_{i\in\mathcal{N}} z_i^* < \sum_{i\in\mathcal{N}} l_i^*$. From the above, we *necessarily* have $p^* = p^b$. Indeed, with $p^* = p^b$, there exists $\boldsymbol{z}^*$ such that a) $z_i^* \le l_i$, and b) $\sum_{i\in\mathcal{N}} z_i^* = \sum_{i\in\mathcal{N}} x_i < l_{\mathcal{N}}$.

Moreover, it is immediate to check that the competitive payoff of RPP i equals $-p^b\left(l_i - x_i\right)$ (cf. (10)).

Case ii) $x_{\mathcal{N}} - l_{\mathcal{N}} > 0$: As result, at the CE, $\sum_{i\in\mathcal{N}} z_i^* > \sum_{i\in\mathcal{N}} l_i^*$. From the above, we *necessarily* have $p^* = p^s$. Indeed, with $p^* = p^s$, there exists $\boldsymbol{z}^*$ such that a) $z_i^* \ge l_i$, and b) $\sum_{i\in\mathcal{N}} z_i^* = \sum_{i\in\mathcal{N}} x_i > l_{\mathcal{N}}$.

Moreover, the competitive payoff of RPP i equals $-p^s\left(l_i - x_i\right)$ (cf. (10)).

Case iii) $x_{\mathcal{N}} - c_{\mathcal{N}} = 0$: In this case, $\forall p^*,\ s.t.\ p^s \le p^* \le p^b$, $z_i^* = l_i, \forall i$ achieves $\sum_{i\in\mathcal{N}} z_i^* = \sum_{i\in\mathcal{N}} x_i = l_{\mathcal{N}}$.

Moreover, the competitive payoff of RPP i equals $-p^*\left(l_i - x_i\right)$ (cf. (10)).

Appendix B
Proof of Lemma 1

Straightforwardly, $C_{\mathcal{S}} \le -v(\mathcal{S})$ because $C_{\mathcal{S}}$ is the minimum possible cost for the subset of prosumers $\mathcal{S}$ after their aggregation. Next, we show that $C_{\mathcal{S}}$ can be achieved by $-v(\mathcal{S})$, i.e., $-v(\mathcal{S}) \le C_{\mathcal{S}}$.

We define $\mathcal{S}^+ \triangleq \{i \in \mathcal{S} \mid x_i - l_i \ge 0\}$ and $\mathcal{S}^- \triangleq \{i \in \mathcal{T} \mid x_i - l_i < 0\}$. The intuition of a redistribution $\{z_i\}$ to achieve $C_{\mathcal{S}}$ is the following: We give as much of the *excess* solar power of the prosumers in $\mathcal{S}^+$ as possible to the prosumers in $\mathcal{S}^-$ to serve their net loads.

Specifically, first consider the case of $x_{\mathcal{S}} - l_{\mathcal{S}} < 0$, i.e.,

$\sum_{i\in\mathcal{S}^-}(l_i - x_i) > \sum_{i\in\mathcal{S}^+}(x_i - l_i)$, we let

$$\forall i \in \mathcal{S}^+,\ z_i = l_i, \tag{24}$$

$$\forall i \in \mathcal{S}^-,\ x_i \le z_i \le l_i,$$

$$\text{so that } \sum_{i\in\mathcal{S}^-}(z_i - x_i) = \sum_{i\in\mathcal{S}^+}(x_i - z_i)\,. \tag{25}$$

As a result,

$$\begin{aligned}
\sum_{i\in\mathcal{S}} f_i(z_i) &= \sum_{i\in\mathcal{S}^+} f_i(z_i) + \sum_{i\in\mathcal{S}^-} f_i(z_i) \\
&= 0 + \sum_{i\in\mathcal{S}^-}\left(-p^b(l_i - z_i)\right) \\
&= -p^b \sum_{i\in\mathcal{S}^-}\left((l_i - x_i) - (z_i - x_i)\right) \\
&= -p^b \left(\sum_{i\in\mathcal{S}^-}(l_i - x_i) - \sum_{i\in\mathcal{S}^+}(x_i - l_i)\right) && (26) \\
&= -p^b(l_{\mathcal{S}} - x_{\mathcal{S}}) = -C_{\mathcal{S}}, && (27)
\end{aligned}$$

where (26) is implied by (24) and (25), and (27) by $x_{\mathcal{S}} - l_{\mathcal{S}} < 0$.

The case of $x_{\mathcal{S}} - c_{\mathcal{S}} \ge 0$ can be proved similarly.

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