**Problem 1.**

An AC generator with an internal resistance of 50 Ω is connected to a step-down transformer loaded with a 1 Ω resistor (see the figure below). Find the voltage across the primary if $N_1 = 50$, $N_2 = 10$ and the open circuit voltage of the generator is 1 V (peak-to-peak).

![AC Generator Circuit Diagram](image)

**Solutions:**

Input impedance of the transformer: $R_{\text{tr}} = (N_1 / N_2)^2 R_L = 25$ Ω.

Voltage across the primary: $V = V_{\text{open}} R_{\text{tr}} / (R_{\text{tr}} + R_{\text{in}}) = 1/3$ (V).
Problem 2.

A diode is biased in the forward direction. Find the voltage drop across the diode if the current is 1 mA. Assume the following parameters:

- saturation current density $J_s = 10^{-10}$ A/cm$^2$;
- cross–section area $A = 1$ mm$^2$ (10mm=1cm)
- non-ideality factor $n = 1.2$.

Solution:

\[
A = 1 \text{mm}^2 = 10^{-2} \text{cm}^2
\]

\[
I_s = J_s \times A = 10^{-12} \text{A}
\]

\[
I = I_s \left( \exp \left( V / nV_{th} \right) - 1 \right) \approx I_s \times \exp \left( V / nV_{th} \right)
\]

\[
V = nV_{th} \times \ln \left( I / I_s \right) \approx 1.2 \times 0.026V \times \ln(10^9) \approx 1.2 \times 0.026V \times 20.7 \approx 0.65V
\]
Problem 3.

The reverse-saturation currents of a Schottky diode and a pn junction diode at 300K are $5 \times 10^{-8}$A and $10^{-12}$A, respectively. The diodes are connected in parallel and are driven by a constant current of 0.5mA. Determine the current and current density in each diode and voltage across each diode. Assume that pn junction diode has nonideality factor equal to 1. Cross-sectional area of pn junction diode is 0.2cm$^{-2}$ and of Schottky diode is 1cm$^{-2}$.

Solution:

Since diodes are connected in parallel they should have the same voltage drop and share total current of 0.5mA. Then: $I_{pn} + I_{Schottky} = 0.5mA$

Using diode IV equation we can write that: $\frac{I_{pn}}{I_{S}} = \ln\left(\frac{I_{Schottky}}{I_{S}}\right)$.

Hence $I_{Schottky} = 0.49999mA$ and $I_{pn} = 0.00001mA$. Voltage is $V = V_{th} \cdot \ln\left(\frac{I_{pn}}{I_{S}}\right) \approx 0.239V$.

Current densities: pn junction diode $\approx 5 \times 10^{-8}$A/cm$^{2}$; Schottky $\approx 5 \times 10^{-4}$A/cm$^{2}$.

Problem 4.

Input signal is sinusoid with 30V peak-to-peak voltage. Sketch the waveforms of current in diode and voltage across it. Find the peak values of the current through and voltage across diode.

Solution:

Diode conducts only when forward bias voltage around 0.7 drops across it. So during the positive half cycle of the input signal the diode conducts only when input voltage is above 10.7V. The peak value of the diode current is $(15V - 10.7V)/100Ohm = 43mA$. 
During negative half cycle of the input signal diode is under reverse bias and current in circuit is just diode reverse saturation current (nearly zero). Peak values of the voltage across diode are: +0.7 V for positive half cycle and -25 V for negative one.

Problem 5.
Determine the bias point (I_{CQ} and V_{CEQ}) for the circuit below. Assume \( \beta = 100 \). For simplicity, neglect base current for bias point calculations. Determine the voltage gain for output signal taken from collector and input applied to base terminal (assume infinite transistor output resistance). Determine input resistance for this amplifier. Determine the voltage gain and input resistance when \( R_E \) is shunted by bypassing capacitor for AC signal. Sketch DC and AC load lines.

Solution:
Voltage at BJT base with respect to ground can be found from voltage divider (this is true only when base current can be neglected otherwise Thevenin’s theorem should be used to convert \( R_1 \) and \( R_2 \) into equivalent resistor connected between equivalent voltage supply and base terminal): \( V_B = \frac{R_2}{R_1 + R_2} \cdot 10V = 2V \). Emitter current can be found assuming that 0.7 V drops between base and emitter for transistor to operate in forward active:

\[
I_E = \frac{2V - 0.7V}{R_E} = 1.3mA \quad \text{and} \quad I_{CQ} \approx I_E = 1.3mA.
\]
Base current can be estimated as $I_B = \frac{I_C}{\beta} \approx 13\mu A$ and indeed it is very small and can be neglected when compared with $\sim 1mA$ flowing through R1 and R2.

Voltage drop between collector and emitter can be estimated as:

$$V_{CE} \approx 10V - 4.3k\Omega \times 1.3mA \approx 5.59V.$$  

To determine voltage gain we observe that input voltage when applied to base terminal is distributed between $R_E$ and $r_\pi$, hence $v_i = i_B \cdot r_\pi + i_E \cdot R_E$. The output signal $v_o = -i_C \cdot R_C$. Hence the voltage gain is: 

$$A_v = \frac{v_o}{v_i} = -\frac{i_C \cdot R_C}{i_B \cdot r_\pi + i_E \cdot R_E} \approx -\frac{R_C}{r_\pi + R_E}.$$

We can estimate $r_\pi$ from 

$$r_\pi = \frac{V_{BE}}{I_B} = \frac{V_{th}}{I_B} \approx \frac{0.026V}{13\mu A} \approx 2k\Omega.$$  

Since $r_\pi \approx 20\Omega$ one can write that $A_v \approx -\frac{R_C}{R_E} = -3.3$.

Circuit input resistance will be composed of R1, R2 and $r_\pi + \beta R_E$ connected in parallel, hence $R_{in} \approx 1.6k\Omega$.

For the case when $R_E$ is shunted for AC signal we have to replace $R_E$ by short circuit in gain and input resistance calculation, hence $A_v \approx -\frac{R_C}{r_\pi} = -165$ and $R_{in} \approx 0.9k\Omega$.

Problem 6.

A MOS differential pair operated at bias current of 0.8mA (current through RSS) employs transistors with $W/L=100$ and $\mu_n \cdot C_{OX}=0.2mA/V^2$, using $R_D=5k\Omega$ and $R_{SS}=25k\Omega$.

(a) Find the differential gain, the common mode gain, and CMRR if the output is taken single-endedly and the circuit is perfectly matched. (b) Repeat (a) when the output is taken differentially.
Solution:

(a) Differential gain $A_D = \left| \frac{V_{O1}}{V_{ID}} \right| = \left| \frac{V_{O2}}{V_{ID}} \right| = g_m \cdot \frac{R_D}{2}$. MOSFET transconductance in saturation can be found from:

$$I_{DS} = \frac{W}{L} \cdot C_{OX} \cdot \mu \cdot \left[ V_{GS} - V_T \right]^2 = \frac{W}{L} \cdot C_{OX} \cdot \mu \cdot \frac{V_{OV}^2}{2};$$

$$g_m = \frac{dI_{DS}}{dV_{GS}} = \frac{W}{L} \cdot C_{OX} \cdot \mu \cdot V_{OV} = \frac{2 \cdot I_{DS}}{V_{OV}}$$

Overdrive voltage can be estimated from $\frac{0.8mA}{2} = 100 \cdot \frac{0.2 \text{ mA}}{V^2} \cdot \frac{V_{OV}^2}{2}$,

hence $V_{OV} \approx 0.2V$. Transconductance is $g_m = \frac{2 \cdot 0.4 \text{ mA}}{0.2V} \approx \frac{4 \text{ mA}}{V}$. Single ended differential gain is $A_D = 10V/V$.

Common mode gain $A_c = \left| \frac{V_{O1}}{V_{IC}} \right| = \left| \frac{V_{O2}}{V_{IC}} \right| = \frac{R_D}{2 \cdot R_{SS}} = \frac{5}{2 \cdot 25} = 0.1V/V$.

CMRR=$A_D/A_c=100$. In dB CMRR=40dB.

(b) $A_D$ will be twice larger, i.e. $A_D = 20V/V$. For fully differential output for perfectly symmetric circuit $A_c=0$, hence CMRR is infinite.
Problem 7:
Find the voltage gain for the circuit below.

\[
\begin{align*}
\text{Solution:} & \\
\text{Net voltage gain is equal to product of the voltage gains of the individual cascades} & \\
A_v &= \frac{V_{out}}{V_{in}} = A_{v1} \cdot A_{v2} = \left( -\frac{20}{5} \cdot 1 + \frac{10}{4} \right) = -14 \frac{V}{V}.
\end{align*}
\]

Problem 8:
Find the voltage gain for the circuit below.

\[
\begin{align*}
\text{Solution:} & \\
\text{Using virtual ground concept:} & \\
\frac{V_{in}}{R} &= -I_s \cdot \left( \exp \left( \frac{V_{out}}{V_{th}} \right) - 1 \right). \\
\text{Hence} & \\
V_{out} &= V_{th} \cdot \left[ \ln \left( \frac{V_{in}}{I_s \cdot R} \right) + 1 \right].
\end{align*}
\]